Test 3, Version Pink,

1(e), 2(d), 3(d), 4(a), 5(e), 6(c), 7(b), 8(**d**), 9(a), 10(b)

Problem 11.

(a) [10pts] By changing to polar coordinates, evaluate the integral

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\sqrt{x^2+y^2}}dxdy.$$

Sol. Polar coordinate

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\sqrt{x^2+y^2}}dxdy = \int_{\theta=0}^{2\pi}\int_{r=0}^{\infty} e^{-r}rdrd\theta = 2\pi\Big(-(r+1)e^{-r}\Big)\Big|_{r=0}^{\infty} = 2\pi$$

(b) [10pts] Evaluate $\iint_A 3xy\,\mathrm{d}x\mathrm{d}y$ where A is the triangle with vertices $(0,0),(3,0),(1,2)$.

Sol.

$$\iint_A 3xy\,\mathrm{d}x\mathrm{d}y = \int_{y=0}^{2}\int_{x=y/2}^{3-y} 3xydxdy = \int_{y=0}^{2}\frac{3}{2}yx^2\Big|_{x=y/2}^{3-y}dy = \int_{y=0}^{2}\frac{3}{2}y\Big((3-y)^2-\frac{y^2}{4}\Big)dy = \frac{15}{2}$$

Problem 12.

(a) [10pts] By changing the order of integration, evaluate the double integral

$$\int_{y=0}^{1}\int_{x=y^2}^{1}\frac{e^x}{\sqrt{x}}dxdy.$$

Sol.

$$\int_{y=0}^{1}\int_{x=y^2}^{1}\frac{e^x}{\sqrt{x}}dxdy = \int_{x=0}^{1}\int_{y=0}^{\sqrt{x}}\frac{e^x}{\sqrt{x}}dydx = \int_0^1 e^x dx = e-1.$$

(b) [10pts] Find the area of the surface $2z = x^2+y^2$ over the unit disc $x^2+y^2<1$.

Sol.

$$A = \iint \sqrt{1+x^2+y^2}dxdy = \int_{\theta=0}^{2\pi}\int_{r=0}^{1}\sqrt{1+r^2}rdrd\theta = 2\pi\,\frac{1}{3}(1+r^2)^{3/2}\Big|_{r=0}^{1} = \frac{2\pi}{3}(\sqrt{8}-1).$$