Sample Test 2

1a, 2e, 3d, 4b, 5e, 6d, 7b, 8a, 9e, 10a

Problem 11
Let $z = \sqrt{x^2 + y^2}$.

(a) Find the differential dz

Sol.

$$dz = \frac{x}{\sqrt{x^2 + y^2}}dx + \frac{y}{\sqrt{x^2 + y^2}}dy$$

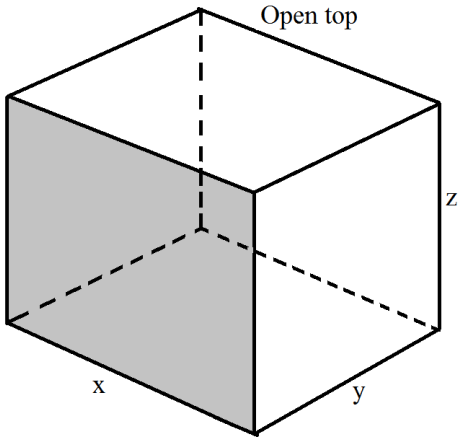
(b) Using differential to approximate value of $\sqrt{(3 + w)^2 + (4 - 2w)^2}$ for small w ($|w| < 0.1$) using the result from (a)

Sol. $x = 3, y = 4$, $dx = w$, $dy = -2w$

$$\begin{aligned}\sqrt{(3 + w)^2 + (4 - 2w)^2} &= \sqrt{3^2 + 4^2} + \frac{x}{\sqrt{x^2 + y^2}}dx + \frac{y}{\sqrt{x^2 + y^2}}dy \\ &= 5 + \frac{3}{5}w + \frac{4}{5}(-2w) \\ &= 5 - w\end{aligned}$$

Problem 12,

A holding tank with **open top** is to be constructed to have a volume of 1 cube meters (i.e., $xyz = 1$). The bottom and the 3 sides are to be constructed with steel costing \$100 per square meter. One final side for viewing is to be constructed with glass costing \$300 per square meter.



Front side (shaded)
Area = xz
Cost: \$300 per square meter

All 3 other sides and bottom
Cost: \$100 per square meter

(a) Write the formula for the cost function $C(x, y, z)$. Then eliminate the variable z to have a function of x, y only.

Sol. Since $xyz = 1$, $xz = \dfrac{1}{y}$ and $yz = \dfrac{1}{x}$

$$C(x, y, z) = 300xz + 100(2yz + xy + xz) = 400xz + 200yz + 100xy = \frac{400}{y} + \frac{200}{x} + 100xy$$

(b) Find the value x, y and z that minimize the cost (Do not use 2nd derivative test).

$$\frac{\partial C}{\partial x} = -\frac{200}{x^2} + 100y = 0, \quad \frac{\partial C}{\partial y} = -\frac{400}{y^2} + 100x = 0$$

Solving these equations we get $y = 2, x = 1$, and so $z = \dfrac{1}{xy} = \dfrac{1}{2}$.

Problem 13,
Compute $\int_{-\pi}^{\pi} \cos x \cdot \cos 2x \; dx$
Show all your work.

Sol.

$$\int_{-\pi}^{\pi} \cos x \cdot \cos 2x \; dx = \int_{-\pi}^{\pi} \frac{e^{ix} + e^{-ix}}{2} \; \frac{e^{i2x} + e^{-i2x}}{2} dx = \frac{1}{4} \int_{-\pi}^{\pi} (e^{i3x} + e^{ix} + e^{-ix} + e^{-i3x}) dx$$

This evaluates to 0 after finding anti-derivatives and plug in the upper and lower limit.