Name: ______________________________

Instructor: Sample Test 3, ACMS 20550

This sample test does not cover all materials in our test

Multiple Choice

1.(6 pts.)Let $\mathbf{A}$ and $\mathbf{B}$ be two non-zero vectors in 3-dimensional spaces. If their scalar product (dot product) satisfies $\mathbf{A} \cdot \mathbf{B} = \mathbf{0}$, then

(a) The angle between $\mathbf{A}$ and $\mathbf{B}$ is $\pi/8$

(b) The angle between $\mathbf{A}$ and $\mathbf{B}$ is $\pi/4$

(c) $\mathbf{A} \times \mathbf{B}$ must be a zero vector

(d) $\mathbf{A}$ and $\mathbf{B}$ are parallel to each other

(e) $\mathbf{A}$ and $\mathbf{B}$ are perpendicular to each other

2.(6 pts.) Let the position vector (with its tail at the origin) of a moving particle be $\mathbf{r} = \mathbf{r}(t) = \sin \frac{\pi}{2} t \,\mathbf{i} - \cos \frac{\pi}{2} t \,\mathbf{j} + 3t \,\mathbf{k}$. Find the velocity vector and the speed at $t = 1$.

(a) velocity vector $\frac{\pi}{2}\mathbf{i} + 3\mathbf{k}$ and speed $\sqrt{\frac{\pi^2}{4} + 9}$

(b) velocity vector $\frac{\pi}{2}\mathbf{j} + 3\mathbf{k}$ and speed $\sqrt{\frac{\pi^2}{4} + 9}$

(c) velocity vector $\mathbf{j} + 3\mathbf{k}$ and speed $\sqrt{10}$

(d) velocity vector $\mathbf{i} + 3\mathbf{k}$ and speed $\sqrt{10}$

(e) velocity vector $\frac{\pi}{2}\mathbf{i} - \frac{\pi}{2}\mathbf{j} + 3\mathbf{k}$ and speed $\sqrt{\frac{\pi^2}{2} + 9}$

Name: ______________________________

Instructor: Sample Test 3, ACMS 20550

3.(6 pts.) If $\phi = z\sin x + y^2$, find the gradient of ϕ at $(0,1,2)$. The gradient is

(a) $(2,2,0)$ (b) $(0,2,1)$ (c) $(1,2,0)$

(d) $(1,2,1)$ (e) $(2,2,1)$

4.(6 pts.) If a sound wave is represented by $p(t) = \sum_{n=1}^{\infty} \frac{\sin 101nt}{90(n-5)^2+1}$, what is the apparent frequency (the frequency you can hear)?

(a) $\frac{303}{2\pi}$ (b) $\frac{101}{2\pi}$ (c) $\frac{707}{2\pi}$

(d) $\frac{505}{2\pi}$ (e) $\frac{404}{2\pi}$

Name: ____________________________

Instructor: Sample Test 3, ACMS 20550

5.(6 pts.) Find the tangent plane of the surface $x^2 + y^2 + z^4 = 3$ at the point $(1, 1, 1)$. The tangent plane is

(a) $2(x-1) + 2(y-1) + (z-1) = 0$

(b) $(x-1) + (y-1) + 2(z-1) = 0$

(c) $(x-1) + (y-1) + (z-1) = 0$

(d) $(x-1) + (y-1) + 4(z-1) = 0$

(e) $(x-1) + (y-1) + \frac{1}{2}(z-1) = 0$

6.(6 pts.) If $f(x)$ is an even function and $g(x)$ is an odd function, then

(a) $f(x) + g(x)$ is always an odd function

(b) $f(x) \cdot g(x)$ is always an even function

(c) $\dfrac{f(x)}{g(x)}$ is always an even function

(d) $f(x) + g(x)$ is always an even function

(e) $f(x) \cdot g(x)$ is always an odd function

Name: ______________________________

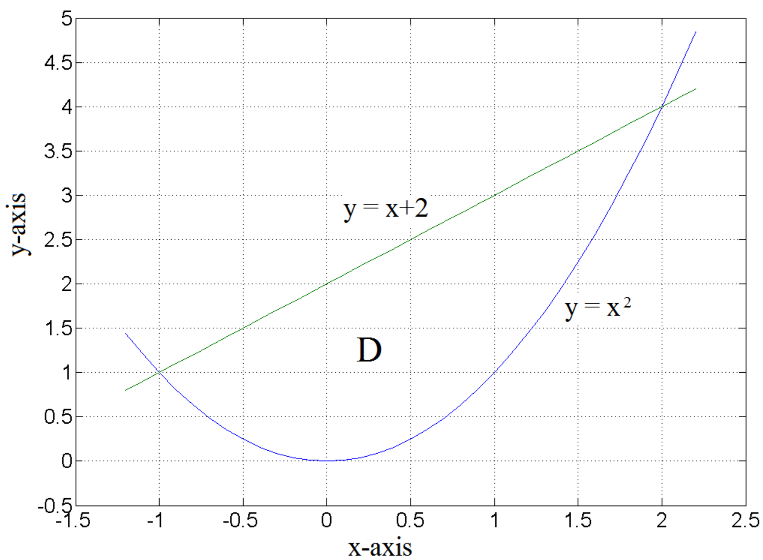
Instructor: Sample Test 3, ACMS 20550

7.(6 pts.) Find the directional derivative of $xy^2 + z^5$ at the point $(1, 1, 1)$ in the direction of $(1, -1, 1)$

(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{8}{\sqrt{3}}$ (c) $\frac{5}{\sqrt{3}}$

(d) $\frac{2}{\sqrt{3}}$ (e) $\frac{1}{\sqrt{3}}$

8.(6 pts.) For the area D in the picture below, which of the following represents the integral $\iint_D f(x,y)dxdy$?



(a) $\int_{x=-1}^{2} \left(\int_{y=x^2}^{y-2} f(x,y)dy \right) dx$ (b) $\int_{y=1}^{4} \left(\int_{x=y-2}^{\sqrt{y}} f(x,y)dx \right) dy$

(c) $\int_{x=-1}^{2} \left(\int_{y=x^2}^{x+2} f(x,y)dy \right) dx$ (d) $\int_{y=0}^{4} \left(\int_{x=y-2}^{\sqrt{y}} f(x,y)dx \right) dy$

(e) $\int_{x=-1}^{2} \left(\int_{y=x+2}^{x^2} f(x,y)dy \right) dx$

Name: ______________________________

Instructor: Sample Test 3, ACMS 20550

9.(6 pts.) Which of the following represents the line integral $\int_C g(x)dy + f(y)dx$, where C is the unit circle $x^2 + y^2 = 1$, oriented counterclockwise.

(a) $\int_0^{2\pi} \Big(-g(\cos\theta)\cos\theta + f(\sin\theta)\sin\theta \Big) d\theta$

(b) $\int_0^{2\pi} \Big(g(\cos\theta)\sin\theta - f(\sin\theta)\cos\theta \Big) d\theta$

(c) $\int_0^{2\pi} \Big(g(\cos\theta)\sin\theta + f(\sin\theta)\cos\theta \Big) d\theta$

(d) $\int_0^{2\pi} \Big(g(\cos\theta)\cos\theta - f(\sin\theta)\sin\theta \Big) d\theta$

(e) $\int_0^{2\pi} \Big(g(\cos\theta)\cos\theta + f(\sin\theta)\sin\theta \Big) d\theta$

10.(6 pts.) Find the area of the surface $2z = x^2 + y^2$ over the unit disc $x^2 + y^2 < 1$.

(a) (2π/3)*(-1+√8) (b) 0 (c) $4\pi/3$

(d) $8\pi/3$ (e) $2\pi/3$

Name: ______________________________

Instructor: Sample Test 3, ACMS 20550

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(14 pts.)

(a) State the divergence theorem.

(b) Compute $\iint_D \mathbf{V} \cdot \mathbf{n}\, d\sigma$, where D is the ellipsoid surface $\frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9} = 1$, the vector $\mathbf{n}$ is the exterior normal vector, and $\mathbf{V} = y^2\mathbf{i} + y\mathbf{j} - z\mathbf{k}$.

Name: ______________________________

Instructor: Sample Test 3, ACMS 20550

Partial Credit

You must show your work on the partial credit problems to receive credit!

12.(13 pts.)

(a) If $f(x)$ is represented by a Fourier Sine Series

$$f(x) = \sum_{n=1}^{\infty} a_n \sin(n\pi x), \quad -\pi < x < \pi$$

what is the formula for a_n?

(b) Compute the Fourier Sine series for the function

$$f(x) = \begin{cases} 2 & 0 < x < \pi \\ -2 & -\pi < x < 0 \end{cases}$$

13.(13 pts.)
Using appropriate coordinate system, evaluate the integral

$$\iint_D e^{-x^2-y^2}\,dxdy,$$

where D is the unit solid disk. Show all your work.

Formula Sheet

1. Surface integral on D represented by $\phi(x,y,z)=0$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \frac{\sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2}}{\left|\frac{\partial\phi}{\partial z}\right|}$$

Surface integral on D represented by $z=f(x,y)$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}$$

2. Cross product

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$$

3. Volume generated by the vectors $\vec{A},\vec{B},\vec{C}$.

If $\vec{A}=(A_1,A_2,A_3),\ \vec{B}=(B_1,B_2,b_3),\ \vec{C}=(C_1,C_2,C_3)$, then

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

4. Polar coordinate system $(r\geq 0,\ 0\leq\theta\leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}, \quad dxdy = rdrd\theta$$

5. Cylindrical coordinate system $(r\geq 0,\ 0\leq\theta\leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}, \quad dxdydz = rdrd\theta dz$$

6. Spherical coordinate system $(r\geq 0,\ 0\leq\theta\leq\pi,\ 0\leq\phi\leq 2\pi)$

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}, \quad dxdydz = r^2\sin\theta drd\theta d\phi$$