Table of Laplace Transforms

	$y = f(t),\ t > 0$ $[y = f(t) = 0,\ t < 0]$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\, dt$	
$L1$	1	$\dfrac{1}{p}$	$\text{Re } p > 0$
$L2$	e^{-at}	$\dfrac{1}{p+a}$	$\text{Re } (p+a) > 0$
$L3$	$\sin at$	$\dfrac{a}{p^2+a^2}$	$\text{Re } p > \lvert \text{Im } a \rvert$
$L4$	$\cos at$	$\dfrac{p}{p^2+a^2}$	$\text{Re } p > \lvert \text{Im } a \rvert$
$L5$	$t^k,\ k > -1$	$\dfrac{k!}{p^{k+1}}$ or $\dfrac{\Gamma(k+1)}{p^{k+1}}$	$\text{Re } p > 0$
$L6$	$t^k e^{-at},\ k > -1$	$\dfrac{k!}{(p+a)^{k+1}}$ or $\dfrac{\Gamma(k+1)}{(p+a)^{k+1}}$	$\text{Re } (p+a) > 0$
$L7$	$\dfrac{e^{-at} - e^{-bt}}{b-a}$	$\dfrac{1}{(p+a)(p+b)}$	$\text{Re } (p+a) > 0$ $\text{Re } (p+b) > 0$
$L8$	$\dfrac{ae^{-at} - be^{-bt}}{a-b}$	$\dfrac{p}{(p+a)(p+b)}$	$\text{Re } (p+a) > 0$ $\text{Re } (p+b) > 0$
$L9$	$\sinh at$	$\dfrac{a}{p^2-a^2}$	$\text{Re } p > \lvert \text{Re } a \rvert$
$L10$	$\cosh at$	$\dfrac{p}{p^2-a^2}$	$\text{Re } p > \lvert \text{Re } a \rvert$
$L11$	$t \sin at$	$\dfrac{2ap}{(p^2+a^2)^2}$	$\text{Re } p > \lvert \text{Im } a \rvert$
$L12$	$t \cos at$	$\dfrac{p^2-a^2}{(p^2+a^2)^2}$	$\text{Re } p > \lvert \text{Im } a \rvert$
$L13$	$e^{-at} \sin bt$	$\dfrac{b}{(p+a)^2+b^2}$	$\text{Re } (p+a) > \lvert \text{Im } b \rvert$
$L14$	$e^{-at} \cos bt$	$\dfrac{p+a}{(p+a)^2+b^2}$	$\text{Re } (p+a) > \lvert \text{Im } b \rvert$
$L15$	$1 - \cos at$	$\dfrac{a^2}{p(p^2+a^2)}$	$\text{Re } p > \lvert \text{Im } a \rvert$
$L16$	$at - \sin at$	$\dfrac{a^3}{p^2(p^2+a^2)}$	$\text{Re } p > \lvert \text{Im } a \rvert$
$L17$	$\sin at - at \cos at$	$\dfrac{2a^3}{(p^2+a^2)^2}$	$\text{Re } p > \lvert \text{Im } a \rvert$

Table of Laplace Transforms (continued)

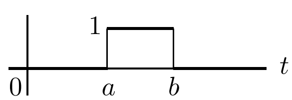
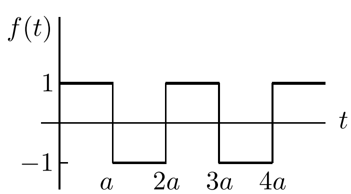
	$y = f(t),\ t > 0$ $[y = f(t) = 0,\ t < 0]$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\, dt$	
$L18$	$e^{-at}(1 - at)$	$\dfrac{p}{(p+a)^2}$	Re $(p+a) > 0$
$L19$	$\dfrac{\sin at}{t}$	$\arctan \dfrac{a}{p}$	Re $p > \lvert \text{Im } a \rvert$
$L20$	$\dfrac{1}{t} \sin at \cos bt$, $a > 0,\ b > 0$	$\dfrac{1}{2}\left(\arctan \dfrac{a+b}{p} + \arctan \dfrac{a-b}{p}\right)$	Re $p > 0$
$L21$	$\dfrac{e^{-at} - e^{-bt}}{t}$	$\ln \dfrac{p+b}{p+a}$	Re $(p+a) > 0$ Re $(p+b) > 0$
$L22$	$1 - \text{erf}\left(\dfrac{a}{2\sqrt{t}}\right), \quad a > 0$ (See Chapter 11, Section 9)	$\dfrac{1}{p}\, e^{-a\sqrt{p}}$	Re $p > 0$
$L23$	$J_0(at)$ (See Chapter 12, Section 12)	$(p^2 + a^2)^{-1/2}$	Re $p > \lvert \text{Im}\, a \rvert$; or Re $p \geq 0$ for real $a \neq 0$
$L24$	$u(t-a) = \begin{cases} 1, & t > a > 0 \\ 0, & t < a \end{cases}$ (unit step, or Heaviside function)	$\dfrac{1}{p}\, e^{-pa}$	Re $p > 0$
$L25$	$f(t) = u(t-a) - u(t-b)$ 	$\dfrac{e^{-ap} - e^{-bp}}{p}$	All p
$L26$		$\dfrac{1}{p} \tanh\left(\frac{1}{2}ap\right)$	Re $p > 0$
$L27$	$\delta(t-a),\ a \geq 0$ (See Section 11)	e^{-pa}	
$L28$	$f(t) = \begin{cases} g(t-a), & t > a > 0 \\ 0, & t < a \end{cases}$ $= g(t-a)u(t-a)$	$e^{-pa}G(p)$ [$G(p)$ means $L(g)$.]	
$L29$	$e^{-at}g(t)$	$G(p+a)$	

Table of Laplace Transforms (continued)

	$y = f(t),\ t > 0$ $[y = f(t) = 0,\ t < 0]$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\, dt$
$L30$	$g(at),\ a > 0$	$\frac{1}{a}\, G\left(\frac{p}{a}\right)$
$L31$	$\frac{g(t)}{t}$ (if integrable)	$\int_p^\infty G(u)\, du$
$L32$	$t^n g(t)$	$(-1)^n \frac{d^n G(p)}{dp^n}$
$L33$	$\int_0^t g(\tau)\, d\tau$	$\frac{1}{p}\, G(p)$
$L34$	$\int_0^t g(t-\tau)h(\tau)\, d\tau = \int_0^t g(\tau)h(t-\tau)\, d\tau$ (convolution of g and h, often written as $g * h$; see Section 10)	$G(p)H(p)$
$L35$	Transforms of derivatives of y (see Section 9): $L(y') = pY - y_0$ $L(y'') = p^2 Y - p y_0 - y_0'$ $L(y''') = p^3 Y - p^2 y_0 - p y_0' - y_0''$, etc. $L(y^{(n)}) = p^n Y - p^{n-1} y_0 - p^{n-2} y_0' - \cdots - y_0^{(n-1)}$	