

Hint for 2.5,

$$\textcircled{1} \quad \forall u, v, w \\ (u|w) + (v|w)$$

$$= \frac{1}{4} [\|u+w\|^2 - \|u-w\|^2 + \|v+w\|^2 - \|v-w\|^2]$$

$$= \frac{1}{4} \left\{ \left[\left\| \frac{u+v}{2} + w + \frac{u-v}{2} \right\|^2 + \left\| \frac{u+v}{2} + w - \frac{u-v}{2} \right\|^2 \right] \right. \\ \left. - \left[\left\| \frac{u+v}{2} - w + \frac{u-v}{2} \right\|^2 + \left\| \frac{u+v}{2} - w - \frac{u-v}{2} \right\|^2 \right] \right\}$$

$$= \frac{1}{4} \left\{ \left[2 \left\| \frac{u+v}{2} + w \right\|^2 + 2 \left\| \frac{u-v}{2} \right\|^2 \right] \right. \\ \left. - \left[2 \left\| \frac{u+v}{2} - w \right\|^2 + 2 \left\| \frac{u-v}{2} \right\|^2 \right] \right\}$$

$$= 2 \left(\frac{u+v}{2} | w \right)$$

\textcircled{2} Let $v=0$

$$(u|w) = 2 \left(\frac{u}{2} | w \right) \quad \forall u$$

so that

$$2 \left(\frac{u+v}{2} | w \right) = (u+v | w).$$

Done!

\textcircled{2} The other parts of the proof require the limit process.