

Math 10250 Activity 3: Polynomial, Rational, and Power Functions (sect. 0.6)

GOAL: To learn the basic properties and behavior of Polynomial, Rational, and Power Functions.

Q1: Write an example of a function which is:

- linear:
- quadratic:
- cubic:
- a 4th degree polynomial:

An n th-degree **polynomial** function is of the form:

$$f(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0,$$

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leading coefficient $a_n \neq 0$

where a_0, a_1, \dots, a_n are constants.

► Behavior of polynomials

Q2: How does the graph of $f(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$ behave as x moves to the left and right?

Tool: If $|x|$ is very large then $a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0 \approx$ _____ .

\uparrow
approximately

Example 1 Match the graphs to the given polynomials $f_1(x)$, $f_2(x)$, $f_3(x)$, $f_4(x)$, and $f_5(x)$. The behavior at infinity is indicated by arrows in the graphs.

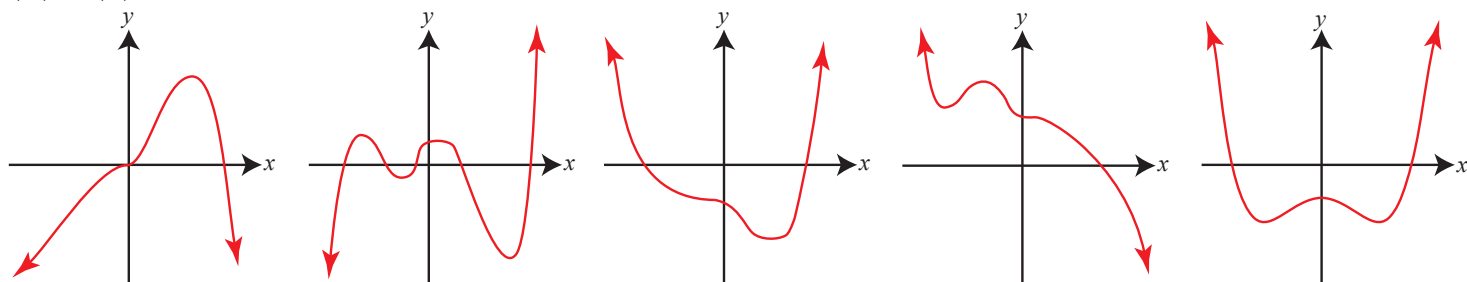
(A) $f_1(x)$ is a 5th degree polynomial with negative leading coefficient.

(B) $f_2(x) = 3x^6 - 4x^2 - 1$

(C) $f_3(x) = 15x^5 - 10x^8$

(D) $f_4(x) = -1 - 4x^3 + 3x^6$

(E) $f_5(x) = x + 3 - 1.04x^3 + 0.04x^5 + 0.12x^4 - 3.12x^2$



► Arithmetic with Polynomial

• polynomial + polynomial = polynomial: Ex. $(2x^2 + 3x) + (x^5 + 1) \stackrel{?}{=}$

• polynomial \times polynomial = polynomial: Ex. $(x^2 + 1)(x^3 - 5x) \stackrel{?}{=}$

Q3: What do we call a $\frac{\text{polynomial}}{\text{polynomial}}$?

A3: $\frac{\text{polynomial}}{\text{polynomial}} =$ _____ .

Example 2 Give the natural domain and the **equations** of all vertical asymptotes of the given rational functions:

a. $\frac{x - 2}{x^2 + 3x + 2}$

b. $\frac{x - 4}{x^2 - 5x + 4}$

Q4: Where are rational functions defined?

A4:

Q5: Where do rational functions have **vertical asymptotes**?

A5:

Example 3 Find a vertical asymptote of $f(x) = \frac{1}{(x - 2)^3}$, if any. If there is a vertical asymptote, determine how the graph behaves (i.e., rises or falls) as it approaches the asymptote from either side by checking whether the function is positive or negative close to the asymptote on either side.

► Power functions

Q6: What do **power functions** look like?

A6: $y = Cx^m$, where C and m are fixed numbers. **Ex.** $y = 2x^5$, $y = 8x^{\frac{4}{3}}$

Q7: How do we interpret x^m when $m = \frac{p}{q}$ for positive integers p and q ?

A7:

$$\boxed{x^{p/q} = \quad = \quad} \quad \boxed{x^{-p/q} = \quad .}$$

Example 4 Compute $(27)^{2/3}$ without using a calculator.

Example 5 Find the natural domain of the power function $f(x) = x^{-3/2}$ and sketch its graph.