Name _____     Date _____

**Math 10250 Activity 5: One-sided and Infinite Limits (sect. 1.1 cont. & sect. 1.2)**

**GOAL:** To learn about the limit of a function $f(x)$ as $x$ approaches to a number $a$ from one side (left or right), get an understanding of infinite limits and relate them to vertical asymptotes.

## ▶ One-sided limits

**Example 1** For the function $y = f(x)$ whose graph is shown in Figure 1, find (by visual inspection) the indicated one-sided limits (if they exist) and determine whether the limit of $f(x)$ exists at the given values of $x$.

(i) $\displaystyle\lim_{x \to -1^-} f(x) \overset{?}{=}$     $\displaystyle\lim_{x \to -1^+} f(x) \overset{?}{=}$     $f(-1) \overset{?}{=}$

      ↑            ↑

    Left-hand limit       Right-hand limit

(ii) $x = 0$

(iii) $x = 1$



Figure 1

(iv) $x = 3$
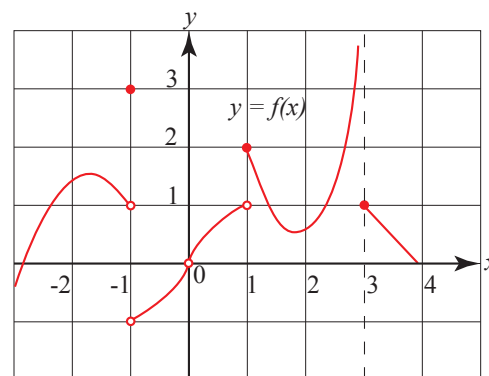
**Fact:** $\boxed{\displaystyle\lim_{x \to a} f(x) = L \quad \text{if and only if}}$

**Example 2** Find $\displaystyle\lim_{t \to 1^+} \frac{t^2 - 1}{\sqrt{t - 1}}$.

**Example 3** If $f(x)$ is the function of example 1 and $g(x) = 8x - 1$ then find the following one-sided limits:

(i) $\displaystyle\lim_{x \to 1^+} [f(x) \cdot g(x)] \overset{?}{=}$

(ii) $\displaystyle\lim_{x \to 1^-} \frac{f(x)}{g(x)} \overset{?}{=}$

1

▶ Explain the meaning of the **infinite limits:**

- $\lim\limits_{x \to a} f(x) = \infty$

- $\lim\limits_{x \to a} f(x) = -\infty$

- $\lim\limits_{x \to a^+} f(x) = \infty \ (\text{or} -\infty)$

- $\lim\limits_{x \to a^-} f(x) = \infty \ (\text{or} -\infty)$

$\boxed{\textbf{Example 4}}$ For the function whose graph is shown in Figure 2 determine its limiting behavior as $x$ approaches each of the points:

   (i) $x = -2$               (ii) $x = 0$             (iii) $x = 2$             (iv) $x = 4$,
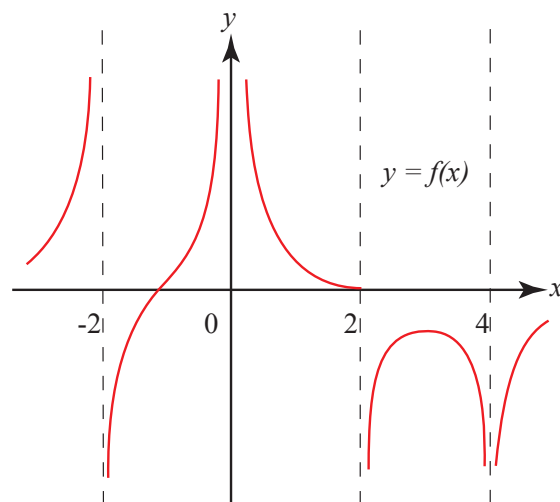
and find its vertical asymptotes.



Figure 2

$\boxed{\textbf{Example 5}}$ $\lim\limits_{t \to 3} \dfrac{1}{(x-3)^2} \overset{?}{=}$

Ans. $\infty$

$\boxed{\textbf{Example 6}}$ $\lim\limits_{x \to 3} \dfrac{x}{x^2 - 9} \overset{?}{=}$

Hint. Check both left and right hand limits.

Ans. DNE

2