

Math 10250 Activity 6: Limits...(sect. 1.2 cont.), and Continuity (sect. 1.3)

GOAL: Understand behavior of functions at $\pm\infty$ and horizontal asymptotes. For rational functions the behavior at $\pm\infty$ is determined by the leading terms.

► Limits at infinity and horizontal asymptotes

• We say that $\lim_{x \rightarrow \infty} f(x) = L$ if \dots

• We say that $\lim_{x \rightarrow -\infty} f(x) = L$ if \dots

• We say that $y = L$ is **horizontal asymptote** if

Example 1 For the function shown in Figure 1 find:

(i) $\lim_{x \rightarrow \infty} f(x) \stackrel{?}{=} \quad$ and $(ii) \lim_{x \rightarrow -\infty} f(x) \stackrel{?}{=} \quad .$

Also, find the horizontal asymptotes.

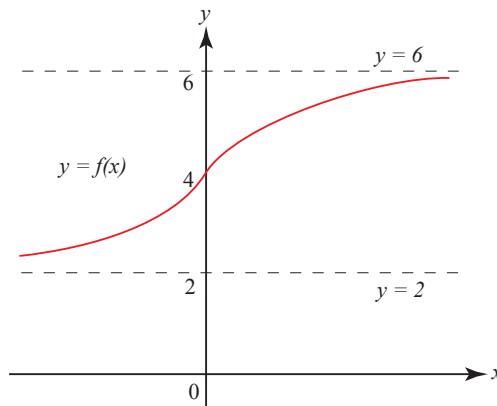


Figure 1

Example 2

(i) $\lim_{x \rightarrow \infty} \frac{x^2 + x}{3x^2 + 7} \stackrel{?}{=} \quad$

(ii) $\lim_{x \rightarrow -\infty} \frac{4x^3 + 7x^2}{x^4 + 2} \stackrel{?}{=} \quad$

(iii) $\lim_{x \rightarrow \infty} \frac{x^3 - 2}{x^2 + 1} \stackrel{?}{=} \quad$

Example 3 A company estimates that when it spends x million dollars to advertise its product, its annual revenue R , in millions of dollars, is modeled by the function $R(x) = 400 - \frac{800}{x + 5}$.

(i) Compute $\lim_{x \rightarrow 0} R(x)$ and $\lim_{x \rightarrow \infty} R(x)$, and draw the graph of $R(x)$. $\lim_{x \rightarrow 0} R(x) = 240$ and $\lim_{x \rightarrow \infty} R(x) = 400$

(ii) If the company is currently spending 35 million on advertising, would you recommend increasing it to 40 million?

► **Idea of Continuity:** A function is continuous if you never have to lift your pencil while drawing its graph. The **discontinuities** are where you have to lift your pencil.

Definition of continuity

A function $f(x)$ is continuous at a point a in its **domain** if

1. $\lim_{x \rightarrow a} f(x)$ _____
2. $\lim_{x \rightarrow a} f(x) \stackrel{?}{=} f(a)$

Example 4 Referring to the function f , whose graph is shown in Figure 2, find all the discontinuities of f in the interval $(-1.2, 7.2)$:

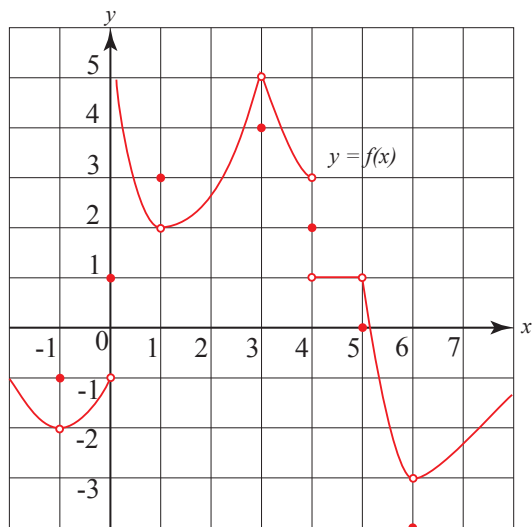


Figure 2