Name _____ Date _____

Math 10250 Activity 7: Continuity (Sec. 1.3)

GOAL: Understand the concept of continuity and its basic properties, including the intermediate value theorem.

Idea of Continuity: A function is **continuous** if you never have to lift your pencil while drawing its graph. The **discontinuities** are where you have to lift your pencil, i.e, at places where there are **gaps** or **holes**.

Example 1 Referring to the function f, whose graph is shown in Figure 1, find all the discontinuities of f in the interval $[-2, 5]$.

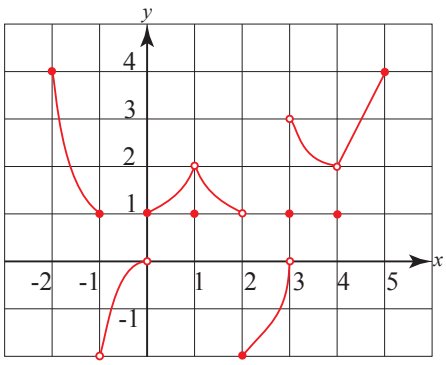

Figure 1

Definition of continuity

A function $f(x)$ is continuous at a point a **in** its **domain** if
1. $\lim_{x \to a} f(x)$
2. $\lim_{x \to a} f(x) =$.

Fact: If f and g are continuous functions at a then

$$\underset{\text{constant}}{c}f(x), \quad f(x)+g(x), \quad f(x)\cdot g(x) \quad \text{and} \quad \frac{f(x)}{g(x)} \text{ where } g(a)\neq 0 \text{ are continuous at } a.$$

From this fact we get:

1. Polynomials are continuous everywhere.
2. $\underset{\text{rational function}}{\frac{\text{polynomial}}{\text{polynomial}}}$ is continuous _____.

Example 2 Determine where the following functions are continuous.

(a) $f(x) = 2x^5 - 3x^2 + 4x - 15$

(b) $f(x) = \dfrac{x^3+1}{x^2+25}$

(c) $f(x) = \dfrac{x^3+1}{x^2-25}$

(d) $f(x) = \begin{cases} \dfrac{x^2-4}{x+2}, & \text{if } x \neq -2 \\ 0, & \text{if } x = -2 \end{cases}$

Example 3 Find the number c that makes $f(x) = \begin{cases} \dfrac{x^3-27}{x-3}, & \text{if } x \neq 3 \\ c, & \text{if } x = 3 \end{cases}$ continuous for every x.

Ans. $c = 27$

▸ The intermediate value theorem and zeros of functions

Intermediate Value Theorem (IVT): If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$ then there is at least one number c in $[a, b]$ such that $f(c) = k$.

Picture:

Existence of Zeros Theorem: Take the above situation where $f(a)$ and $f(b)$ have opposite signs.

Picture:

Then by IVT, there is at least one number c in (a, b) such that $f(c) = 0$. This helps us find zeros of functions (i.e roots).

Example 4 Suppose a continuous function $f(x)$ satisfies the following table of values:

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-2	-3	-2	-1	1	2	1	-1	-2

How many roots can you be sure of $f(x)$ having on the interval $(-4, 4)$, and where they are located.

Example 5 Does the equation $x^4 + 8x^3 - x^2 - 4x - 1 = 0$ have a root inside the interval $(0, 1)$?

Problem Explain why there was a time between the day you were born and today when your height in inches (say 21) was equal to your weight in pounds (say 7).