Name ______________________________ Date ___________________

Math 10250 Activity 8: Exponential Functions (sect. 2.1)

GOAL: Learn exponential functions with different bases and use them to model real-world situtations.

▶ **Exponential functions** are of the form : $\boxed{f(x) = b^x}$, where $b > 0$ is called **base**, like $f(x) = 2^x$.

Q1: Where do they appear?

A1: Everywhere! For example, if we put \$1 in an account paying 5% interest, compounded annually, then t years later it will become $f(t) = (1.05)^t$, which is an **exponential function** with base $b = 1.05$.

▶ The laws of exponents. For $b > 0$ and u and v any numbers, we have

(1) $b^{u+v} \stackrel{?}{=}$; e.g., $2^{3+2} \stackrel{?}{=}$ and $2^3 \cdot 2^2 \stackrel{?}{=}$

(2) $b^{u-v} \stackrel{?}{=}$; e.g., $2^{3-2} \stackrel{?}{=}$ and $\dfrac{2^3}{2^2} \stackrel{?}{=}$

(3) $b^{ru} \stackrel{?}{=}$ for any real number r; e.g., $2^{3\cdot 2} \stackrel{?}{=}$ and $(2^2)^3 \stackrel{?}{=}$

(4) $b^0 \stackrel{?}{=}$

(5) $b^{-v} \stackrel{?}{=}$; e.g., $2^{-2} \stackrel{?}{=}$

Example 1 If $b^u = 2$ and $b^v = 3$ then $b^{u-v} \stackrel{?}{=}$

▶ Graph of $y = b^x$

Case 1: $b > 1$

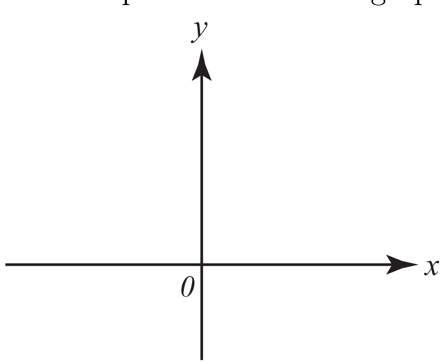
For example, $y = 2^x$.

(i) Complete the table below:

x	-1	-0.5	0	0.5	1
2^x	0.5		1		2

Truncate answers to 2 decimal places

(ii) Plot the points and sketch graph:



(iii) **Properties of b^x when $b > 1$:**

- $b^0 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- $\lim_{x\to-\infty} b^x \stackrel{?}{=}$ $\lim_{x\to\infty} b^x \stackrel{?}{=}$
- Asymptote:

Case 2: $0 < b < 1$

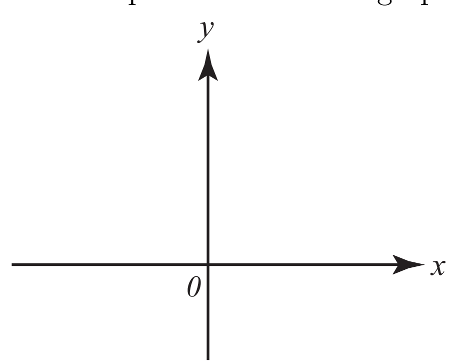
For example, $y = (1/2)^x$.

(i) Complete the table below:

x	-1	-0.5	0	0.5	1
$(1/2)^x$	2		1		0.5

Truncate answers to 2 decimal places

(ii) Plot the points and sketch graph:



(iii) **Properties of b^x when $0 < b < 1$:**

- $b^0 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- $\lim_{x\to-\infty} b^x \stackrel{?}{=}$ $\lim_{x\to\infty} b^x \stackrel{?}{=}$
- Asymptote:

▶ Three applications of the exponential function

1 Compound interest

Example 1 If \$1,000 is invested in an account paying 5% interest, how much will it grow to in 10 year if the interest is compounded monthly?

- Annual rate $= r \stackrel{?}{=}$ (in **decimals**)
- Compounding per year $= n \stackrel{?}{=}$
- Compounding rate $= \frac{r}{n} \stackrel{?}{=}$
- Time $= t \stackrel{?}{=}$ (in **years**)

At the end of 1st period have: ____________________________

At the end of 2nd period have: ____________________________

At the end of 3th period have: ____________________________

⋮

At the end of nth period have: ____________________________

Interest compounded 12 times a year over t **years**

At the end of 1 year (12 periods) have: ____________________________

At the end of 2 years (24 periods) have: ____________________________

⋮

At the end of t years have: ____________________________

General formula:

$$\boxed{A(t) = P\left(1 + \frac{r}{n}\right)^{tn}}$$

Example 2 If \$8,000 is invested in an account paying 3% interest, how much will it grow to in 15 years if the interest is compounded quarterly?

2 Population Growth (with unlimited resources) $\boxed{P(t) = P_0 b^t}$

Example 3 A certain bacteria culture grows exponentially. In 1 hour the population grows from 300,000 to 500,000. Write a formula expressing the population P as a function of the time t in hours.

Ans. $P(t) = 300,000(\frac{5}{3})^t$

3 Decay of radioactive substances: $\boxed{y = y_0 b^t}$

Example 4 Radon gas decays according to the formula $y = y_0(0.835)^t$, where t is measured in days. If there are 500 cubic centimeters left after 7 days, how much was there to begin with?

$y_0 = 500(0.835)^{-7}$