Name _____ Date _____

Math 10250 Activity 10: Logarithmic Functions (Sec. 2.3)

GOAL: Learn logarithmic functions as inverses of exponential functions and use them to model various interesting situations, like intensity of earthquake, noise level, and acidity of beer.

Q1: What "undoes" the exponential function $f(x) = b^x$? (e.g. If $f(x) = 2^x$ then $3 \overset{f}{\mapsto} 2^3 = 8$)

A1: The **logarithmic function with base *b***, denoted $\log_b$. (If $g(x) = \log_2 x$ then $8 \overset{g}{\mapsto} \log_2 8 = 3$)

Definition: $\log_b$ (for $b > 0, b \neq 1$) is defined by

$$\log_b x = y \quad \Leftrightarrow \quad .$$

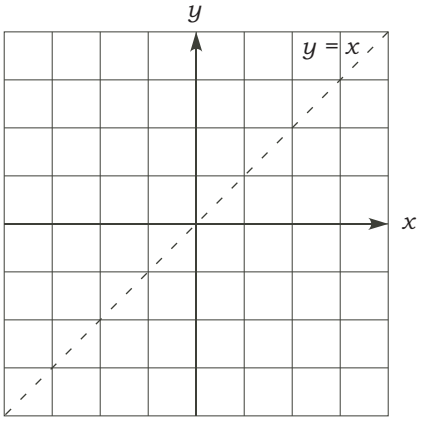
Example 1 Express the following logarithms as an integer or fraction without using a calculator.

(a) $\log_3 9$

(b) $\log_{(0.1)} 1000$

▸ The graph of $\log_b x$ for $b > 1$:

As an example, first graph $y = 2^x$ and obtain the graph of $y = \log_2 x$.



Properties of logarithmic functions

- $\log_b 1 \overset{?}{=}$
- domain $\overset{?}{=}$ and range $\overset{?}{=}$
- It's continuous and increasing.
- $\lim_{x\to\infty} \log_b x \overset{?}{=}$ and $\lim_{x\to 0^+} \log_b x \overset{?}{=}$

Note: The most common choices for b are 10, e and 2.

▸ The laws of logarithms. (*Reversing the laws of exponents*) Let $s, t > 0$. Then

(1) $\log_b(st) \overset{?}{=}$; e.g., $\log_2(3 \cdot 8) \overset{?}{=}$

(2) $\log_b\left(\frac{s}{t}\right) \overset{?}{=}$; e.g., $\log_2\left(\frac{3}{8}\right) \overset{?}{=}$

(3) $\log_b(t^r) \overset{?}{=}$ for any number r; e.g., $\log_2(3^7) \overset{?}{=}$

(4) $\log_b 1 \overset{?}{=}$

(5) $\log_b\left(\frac{1}{t}\right) \overset{?}{=}$; e.g., $\log_2\left(\frac{1}{8}\right) \overset{?}{=}$

Q2: Can you explain property (1)?

A2:

Example 2 Use the approximation $\log_{10} 0.5 \approx -0.301$ to estimate $\log_{10} 20$.

Example 3 Use the approximation $\log_2 3 \approx 1.585$ and $\log_2 5 \approx 2.322$ to estimate $\log_2 45$.

Example 4 Suppose A and b are positive numbers with $\log_3 A = b$. Write $\log_3 \left(\dfrac{3}{\sqrt[3]{A}} \right)$ in terms of b.

Example 5 A bank teller claims that a saving account with principal of \$1000 earning interest at a annual rate of 1.3 %, compounded weekly, after T years would at least double. What is the smallest possible T in whole years? Ans. 54 years.

▸ Logarithms with base 10

Logarithms with base 10, called **common logarithms**, are used in many well-known applications.

1 The Richter scale $\qquad \boxed{\text{Richter value } = \log_{10} \left(\dfrac{x}{A} \right),}$

where A is the amplitude of the seismic wave of a reference earthquake and x is the amplitude of the seismic wave of the earthquake in question.

Example 6 One of the worst earthquakes in history occured in Tokyo and registered 8.3 on the Richter scale. A more recent earthquake in California in 1989 registered 7.2. How much more severe was the earthquake in Tokyo in terms of the amplitude of its seismic wave? Ans. $10^{1.1}$ larger.

2 The decibel scale $\qquad \boxed{\text{Noise level in decibels } = 10 \log_{10} \left(\dfrac{x}{I} \right),}$

where I is the amplitude of a minimal audible sound wave and x is the amplitude of another sound wave. **Read Text Example 2.3.3 (Pg 141).**

3 The pH scale $\qquad \boxed{\text{pH value } = -\log_{10}[\mathrm{H}^+],}$

where $[\mathrm{H}^+]$ is the concentration of hydrogen ions in a solution. **Read Text Example 2.3.4 (Pg 142).**