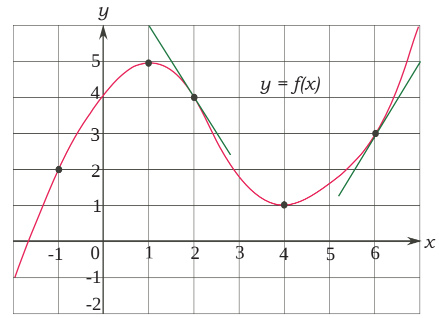
Name ______________________________ Date ____________________

Math 10250 Activity 13: The Derivative of a Function (Sec. 3.2)

GOAL: To understand that the slope of the graph of a function $f(x)$ is dependent on x. The function that gives the slope of the graph of $f(x)$ is called the derivative of $f(x)$. We will also learn about some basic properties of the derivative, and the derivatives of power functions and polynomial.

Example 1 For the function $y = f(x)$ whose graph is shown, compute or estimate the following values:



Slope of the tangent line to the graph of $f(x)$ at $x = 2 \stackrel{?}{=}$

Slope of the graph of $f(x)$ at $x = 4 \stackrel{?}{=}$

Instantaneous rate of change of $f(x)$ at $x = 6 \stackrel{?}{=}$

Rate of change of $f(x)$ at $x = -1 \stackrel{?}{=}$

Remark: The slope of the graph of $f(x)$ or rate of change of $f(x)$ varies according to x. This gives us a new function called the **derivative** of $f(x)$. We denote the derivative of $f(x)$ by $f'(x)$.

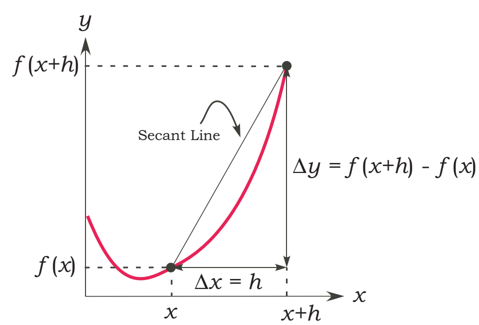
Find the following values for the function in Example 1:

$f'(1) \stackrel{?}{=}$ $\qquad$ $f'(2) \stackrel{?}{=}$ $\qquad$ $f'(4) \stackrel{?}{=}$ $\qquad$ $f'(6) \stackrel{?}{=}$ $\qquad$ $f'(-1) \stackrel{?}{=}$

Difference Quotient and Leibniz's notation

Recall that the limit definition of the rate of change of $f(x)$. Then we have:

Derivative of $f(x) = f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

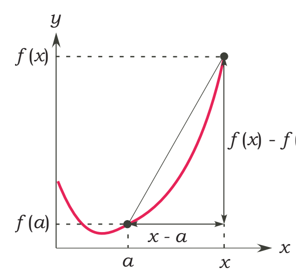


Difference quotient $= \frac{f(x+h)-f(x)}{h} = \dfrac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$

So $f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} =$ ____________

↑ Leibniz's notation

We also write: $\qquad f'(x) = \dfrac{dy}{dx} = \dfrac{d}{dx}[f(x)]$



Other Notations for the Derivative

For each fixed value a in the domain of $f(x)$, we can also write:

$f'(a) =$

Example 2 Suppose that $f(x)$ is a function whose graph goes through the point (1,5) and whose tangent line at that point has the equation $2x + y = 7$. Without computing, find each of the following limits:

(a) $\lim_{h \to 0} \dfrac{f(1+h) - 5}{h} \stackrel{?}{=}$ $\qquad$ (b) $\lim_{\Delta x \to 0} \dfrac{f(1+\Delta x) - 5}{\Delta x} \stackrel{?}{=}$ $\qquad$ (c) $\lim_{x \to 1} \dfrac{f(x) - 5}{x - 1} \stackrel{?}{=}$

Example 3 Use the definition of derivative and no other formula to find $f'(x)$ where $f(x) = \frac{1}{x} = x^{-1}$.

Rules for finding derivatives: (Notations: $f'(x) = (f(x))' = \frac{d}{dx}[f(x)]$)

0. The derivative of a constant is zero: $(c)' \stackrel{?}{=}$. e.g. $(8)' \stackrel{?}{=}$, or $(\sqrt{2})' \stackrel{?}{=}$, or $(e)' \stackrel{?}{=}$

1. The Power Rule $(x^m)' \stackrel{?}{=}$. e.g. $(x^5)' \stackrel{?}{=}$, or $(x^{-0.8})' \stackrel{?}{=}$

2. The Constant Multiple Rule $\frac{d}{dx}[cf(x)] \stackrel{?}{=}$. e.g. $(3x^5)' \stackrel{?}{=}$
   (c: constant)

3. The Sum Rule $\frac{d}{dx}[f(x) + g(x)] \stackrel{?}{=}$. e.g. $(x^2 - 5x + 4)' \stackrel{?}{=}$

Q2: Explain why is Sum Rule true.

A2:

Example 4 $\frac{d}{dx}\left[2x^4 + 3x^{-3} - \frac{\pi}{e}\right] \stackrel{?}{=}$

Example 5 Find the equation of the line tangent to the graph $y = x^3 - 2x$ at $x = 2$.