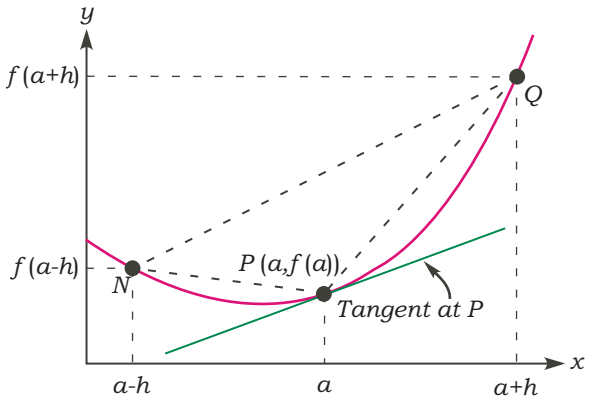
Name _____ Date _____

Math 10250 Activity 14: The Derivative as a Rate (Sec. 3.3)

GOAL: To focus our attention on the interpretation of the derivatives as a rate of change and learn what it represents in different physical contexts. For example, velocity is the derivative of the position function, and the acceleration is the derivative of the velocity function.

Estimating the derivative



- Forward difference formula:

$$f'(a) \approx \text{Slope of chord PQ} = \_\_\_\_\_$$

- Backward difference formula:

$$f'(a) \approx \text{Slope of chord NP} = \_\_\_\_\_$$

- Central difference formula:

$$f'(a) \approx \text{Slope of chord NQ} = \_\_\_\_\_$$

Example 1

x	2.98	2.99	3	3.01	3.02
$f(x)$	7.87	7.95	8	8.06	8.09

Give as many estimates as possible for each of the following derivatives of $f(x)$ with the table above:

(a) $f'(3)$

(b) $f'(2.98)$

(c) $f'(3.02)$

Average and instantaneous velocity

- $s(t)$ = Position of object at time t from some fixed point O.
- Average velocity over the time interval $a \le t \le b = \dfrac{\text{change in position}}{\text{change in time}} = \_\_\_\_\_$

Example 2 A puppy on Douglas Road is 60 meters west of 7-Eleven at 12:00 PM. If the position (in meters) of the puppy is measured from 7-Eleven (origin O) t minutes after 12:00PM is given in Figure 1, answer the following questions about the puppy:

(a) What is its position and distance traveled when $t = 10$?

(b) What is its position and the distance traveled when $t = 80$?

(c) Did the puppy stop for a break? If yes, when and how long?

(d) What is its average velocity for $0 \le t \le 20$?

(e) What is its average velocity between 12:20PM and 1:10PM?

(f) What is its average velocity for $0 \le t \le 80$? What about average speed?

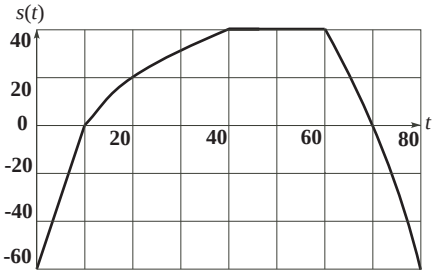


Figure 1

Remarks:

- If average velocity is **positive** then object has moved to the _____.
- If average velocity is **negative** then object has moved to the _____.
- Average speed between $a \leq t \leq b =$ _____

Instantaneous Velocity, Speed, and Acceleration

If $s(t)$ is the position of an object from a fixed point O. Then we define its (instantaneous) velocity, speed and acceleration as follows:

- Instantaneous velocity $v(t)$ = Rate of change of position $\stackrel{?}{=}$

  If $v(t) > 0$ then object is moving to$\cdots$

  If $v(t) < 0$ then object is moving to$\cdots$

- Instantaneous speed = Magnitude of velocity $\stackrel{?}{=}$

- Instantaneous acceleration $a(t)$ = Rate of change of velocity $\stackrel{?}{=}$

  If $a(t) > 0$ then velocity of object is _____.

  If $a(t) < 0$ then velocity of object is _____.

Example 3 A ball is thrown into the air and its height in feet after t seconds is given by $s = -16t^2 + 32t + 48$ until it hits the ground.

(a) Write a formula for the ball's velocity until it hits the ground.

(b) What is its velocity at the end of 1 second? In what direction (up or down) is it moving at the end of 1 second? What about its speed?

(c) What is its velocity at the end of 1.5 seconds? In what direction (up or down) is it moving at the end of 1.5 seconds? What about its speed?

(d) What is the ball's acceleration at the end of 0.5 seconds? What is the ball's acceleration after 1 second?