Name ______________________________ Date ___________________

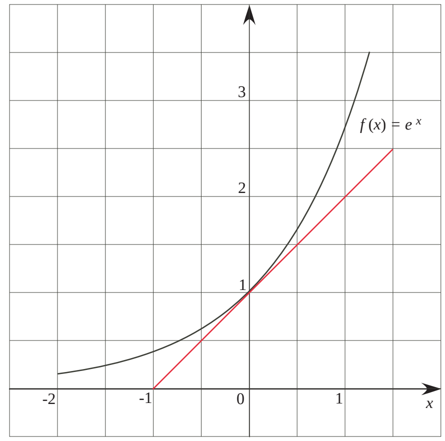
Math 10250 Activity 17: Derivative of Logarithms and Exponentials (Sec. 3.5)

GOAL: To introduce the derivative of Exponential and Logarithmic functions, and use them to model population growth, cooling and radioactive decay.

▶ Derivative Formulas

Q1: What is the derivative of $f(x) = e^x$?

A1: Let's find the derivative of $f(x) = e^x$ at $x = 0$ first.
By definition of derivative:



$$f'(0) = \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{e^h - e^0}{h} = \lim_{h\to 0}\frac{e^h-1}{h} \stackrel{?}{=}$$

Let's estimate the limit above by making a table of values:

h	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01
$\dfrac{e^h-1}{h}$				?			

So $f'(0) \stackrel{?}{=}$ and $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{h\to 0}\frac{e^x\cdot e^h - e^x}{h} \stackrel{?}{=}$

$\boxed{(e^x)' \stackrel{?}{=}}$ ← **derivative of the natural exponential function:** In gereral → $\boxed{(e^{cx})' \stackrel{?}{=}}$

Example 1 If $f(t) = e^t + e^{3t} + \dfrac{t^2 + te^t}{2t}$ then $f'(t) \stackrel{?}{=}$

Since $b^x = e^{(\ln b)x}$, $\boxed{(b^x)' = \qquad\qquad =}$ ← **derivative of exponential function**

Example 2 If $g(x) = \pi^x + x^{2e} + e^{2x} + x^\pi$ then $g'(x) \stackrel{?}{=}$

$\boxed{(\ln x)' \stackrel{?}{=}}$ ← **derivative of the natural logarithm function:**

Example 3 Find the derivative:

(a) $\frac{d}{dx}\left(\frac{1}{x} + \ln x + 13\right)$

(b) $\frac{d}{dx}\left(\ln\left(\frac{x^4}{7}\right)\right)$

Example 4 Find the equation of the line tangent to the graph of $y = 2x^2 + \ln x$ at $x = 1$.

▶ Differential equations as models for exponential growth and decay

Exercise Verify that the **exponential** function $y = Ae^{rt}$ is the solution of the differential equation $\frac{dy}{dt} = ry$ and $y(0) = A$.

We see here that the rate of change of y is proportional to its amount present at time t. Many growth and decay phenomena in nature are modeled by such differential equations. The quantity r is called the continuous *growth* rate if r_____, and the *decay* rate if r_____. Here are some examples:

1 **Modeling Population Growth** (unrestricted resources)

Example 5 The size at the end of t days of a colony of insects is given by the formula $P(t) = 2,000(1.03)^t$. What is the continuous growth rate of the insect colony?

2 **Newton's Law of Cooling:** The rate at which an object cools is proportional to the difference between its temperature and the surrounding temperature.

$\boxed{\frac{dH}{dt} = \qquad\qquad}$ ← **model for Newton's law of cooling.** The solution is → $\boxed{H = M + Ae^{kt}}$

Example 6 A bowl of soup is brought into a room whose temperature is kept at a steady 70°F. Three minutes later the temperature of the soup is 90°F and decreasing at a rate of 4°F per minute. Find its temperature as a function of time.

3 **Carbon Dating** Invented by Willard Libby to determine the age of plant and animal samples. Plants and animals contain a mixture of C-14 (radioactive carbon) and C-12 (nonradioactive carbon). Carbon dating compares the *rates* at which radioactive isotope C-14 is decaying in an ancient sample and a fresh sample to obtain an idea of the age of the ancient sample.

Example 7 In 1950, carbon dating was used to determine the age of wood samples excavated from a city in Babylon. The rate of radioactive carbon decay of these samples was measured at 4.09 disintegrations per minute (dpm). By comparison, the decay rate from fresh wood samples was measured at 6.68 dpm. Using 5,568 years as the half-life of radioactive carbon, estimate the age of the samples.