

**Math 10250 Activity 18: The Product and Quotient Rules (Sec. 3.6)****GOAL:** To learn how to compute the derivatives of a product and a quotient of two functions.

► **The Product Rule:**  $\frac{d}{dx}[f(x) \cdot g(x)] =$  \_\_\_\_\_

Note:  $\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[g(x) \cdot f(x)]$ .

**Example 1** Use the product rule to find the derivatives:

(a)  $\frac{d}{dx}[x^2(3x^3 - x)]$

(b)  $\frac{d}{dx}[e^{-2x} \ln x]$

► **The Quotient Rule:**  $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] =$  \_\_\_\_\_

In general,  $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] \neq \frac{d}{dx} \left[ \frac{g(x)}{f(x)} \right]$ .

**Example 2** Find the equation of the tangent line to the graph of  $y = \frac{x}{x^2 + 1}$  at the point  $x = 2$ .

**Example 3** Use the appropriate differentiation rule you learned so far to find the derivatives below. Some algebra may be helpful.

(a)  $\frac{d}{dx} \left( \frac{x^2 + x - 3}{100} \right)$

(c)  $\frac{d}{dx} \left( \frac{\ln x}{x^2} \right)$

(b)  $\frac{d}{dx} \left( \frac{x + e^x}{e^x} \right)$

(d)  $\frac{d}{dx} \left( \frac{x^2 + x - 3}{x^{10}} \right)$

**Example 4** Suppose the demand for a certain product is given by  $q = f(p)$ , where  $p$  is the price per unit and  $q$  is the number sold. The revenue is given by  $R = pq$ .

(a) If  $f(300) = 20,000$  and  $f'(300) = -30$ , find  $dR/dp$  when  $p = 300$ .

(b) If the product is currently selling for \$300 per unit, should the company increase or decrease the price in order to raise the revenue?

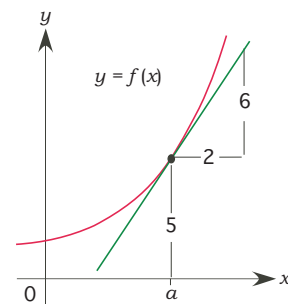
**Example 5** For what  $x$  does the graph  $y = xe^x$  have slope zero?

Ans:  $x = -1$

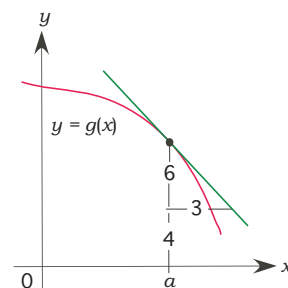
**Example 6** Find the equation of the tangent line to the graph of  $y = \frac{1 - \ln x}{1 + \ln x}$  at  $x = 1$ . Ans:  $y = -2x + 3$

**Example 7** Let  $p(x) = f(x)g(x)$  and  $q(x) = \frac{f(x)}{g(x)}$ . Using the graph of  $f(x)$  and  $g(x)$  find

(a)  $p'(a)$



(b)  $q'(a)$



Ans:  $p'(a) = 20$  and  $q'(a) = 0.4$