Name ______________________________ Date ___________________

Math 10250 Activity 19: The Chain Rule (Sec. 3.7)

GOAL: To learn how to compute the derivative of a composition of two functions.

Q1: What rule would you use to compute the following derivatives:

(a) $\left[\dfrac{x^2+1}{x+1}\right]'$ ___________________

(b) $\left[\dfrac{x^2+1}{e+1}\right]'$ ___________________

(c) $[(x^2+1)\pi]'$ ___________________

(d) $[(x^2+1)\pi^x]'$ ___________________

(e) $[\ln(x^2+1)]'$ ___________________

▶ **The Composite Function.** A function $h(x)$ is said to be a composite function of $g(x)$ followed by $f(x)$ if $h(x) = f(g(x))$. We may write: $h : x \overset{g}{\longmapsto}$ ________ $\overset{f}{\longmapsto}$ ________

Example 1 Find functions $f(x)$ and $g(x)$, not equal x, such that $h(x) = f(g(x))$:

(a) $h(x) = (x^4 + 2x^2 + 7)^{21}$ $h : x \overset{g}{\longmapsto}$ ____________ $\overset{f}{\longmapsto}$ ____________

Ans: $f(x) \overset{?}{=}$ ____________ and $g(x) \overset{?}{=}$ ____________

(b) $h(x) = e^{x^2+1}$ $h : x \longmapsto$ ____________ $\longmapsto$ ____________

Ans: $f(x) \overset{?}{=}$ ____________ and $g(x) \overset{?}{=}$ ____________

▶ **The Chain Rule**

Q2: In a SMS (short message service) competition for the title of "Fastest SMS Thumbs", it is observed that Competitor A inputs text three times faster than B, and Competitor B inputs text two times faster than C. How much faster is Competitor A than Competitor C? Why?

A — — $\overset{3}{\text{Rate of } A \text{ relative to } B}$ — — B — — $\overset{2}{\text{Rate of } B \text{ relative to } C}$ — — C

— — — — — — — — — ? Rate of A relative to C ? — — — — — — — — —

Suppose $y = f(g(x))$. To find a formula for $\dfrac{dy}{dx} = \dfrac{d}{dx}[f(g(x))]$, we set $u = g(x)$ then $y = f(u)$.

y — — $\overset{\frac{dy}{du}}{\text{Rate of } y \text{ relative to } u}$ — — u — — $\overset{\frac{du}{dx}}{\text{Rate of } u \text{ relative to } x}$ — — x

$\dfrac{dy}{dx} \overset{?}{=}$

— — — — — — — — ? Rate of y relative to x ? — — — — — — — — —

So expect $\boxed{\dfrac{dy}{dx} = \qquad\qquad}$

Our guess is in fact correct, and the formula for $\dfrac{dy}{dx}$ is called the **Chain Rule** (in Leibniz notation).

But $\dfrac{dy}{dx} = \dfrac{d}{dx}[f(g(x))] = [f(g(x))]'$, $\dfrac{dy}{du} = f'(u) = f'(g(x))$ and $\dfrac{du}{dx} = g'(x)$. Thus we also have:

$$\boxed{\frac{d}{dx}[f(g(x))] = [f(g(x))]' = \qquad\qquad\qquad}$$

Example 2 Find the derivatives:

(a) $[\ln(x^2+1)]' \stackrel{?}{=}$

(b) $[(x^4+2x^2+7)^{21}]' \stackrel{?}{=}$

(c) $[x\ln(2+e^x)]' \stackrel{?}{=}$

(d) $\left[e^{x^2+1}\right]' \stackrel{?}{=}$

Example 3 For what x does the graph of $y = e^{\frac{1}{3}x^3-4x}$ have slope zero?

Example 4 Let $f(x) = \dfrac{g(x^2)}{\sqrt{x+1}}$. Find the slope of the graph of $f(x)$ at $x = 3$.

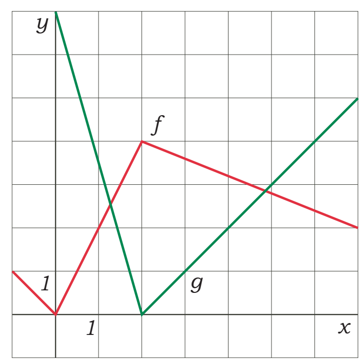
x	$g(x)$	$g'(x)$
3	5	2
4	0	7
9	-2	3

Example 5 Let $A(x) = g(f(x))$ and $B(x) = g(g(x))$. Use the graph of $f(x)$ and $g(x)$ to compute each of the following derivatives if it exists. If it does not exist, explain why.

Ans: One of them does not exist. Why?

(a) $A'(1) \stackrel{?}{=}$

(b) $B'(1) \stackrel{?}{=}$



Example 6 Diatoms are microscopic algae surrounded by a silica shell that are found both in salt and fresh water, and they are a major source of atmospheric oxygen. The size of a diatom colony depends on many factors, including temperature. Suppose that samples taken in a midwestern lake showed that the concentration of diatoms was modeled as a function of temperature by the equation

$$C = 1.4 - e^{-0.001h^2} \quad \text{for } 0 < h < 40,$$

where C is the concentration of diatoms (in millions per cubic centimeter) and h is the temperature of the water (in degrees Celsius).

(a) $\dfrac{dC}{dh} \stackrel{?}{=}$

(b) Suppose the temperature of the lake is 10° and falling at the rate of 2 degrees per hour. At what rate is the concentration of diatoms changing with respect to time? Ans: $-0.04e^{0.1}$