Name ______________________________ Date ___________________

Math 10250 Activity 21: First Derivative Tests (Sec. 4.1)

GOAL: To use information given by $f'(x)$ to find where $f(x)$ is increasing and decreasing, and to locate maxima and minima.

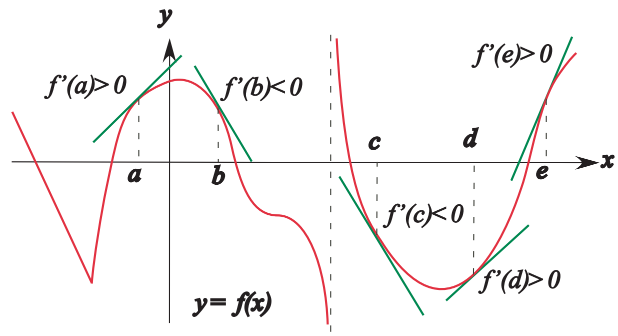


Figure 1

▶ The derivative test for increasing and decreasing functions

Q1: What does f' tell us about f?

A1: • If $f'(x) > 0$ for $\alpha < x < \beta$, then $f(x)$ is ____________ for $\alpha < x < \beta$.

• If $f'(x) < 0$ for $\alpha < x < \beta$, then $f(x)$ is ____________ for $\alpha < x < \beta$.

▶ Determining the sign of $f'(x)$

What we have seen thus far: To find where f is increasing or decreasing, we need to find where $f'(x)$ is positive and negative. To do this, we start by finding the **critical points** of $f(x)$.

Definition: Critical points of the function f are points c in the domain of f where (a) ____________ or (b) ____________ does not exist.

Q2: Where are the critical points of the function $f(x)$ in Figure 1. Label them $c_1, c_2, \cdots$

Remark: The only possible places (of x) where $f'(x)$ changes signs are at (i) **critical points** or at (ii) where the graph has a **vertical asymptote** or undefined.

Example 1 Find all values of x for which $f(x) = x^3 + 3x^2 - 9x + 3$ is increasing or decreasing with the steps outlined below.

Step 1: Find all **critical points** of f. (That is all points c in the domain where $f'(c) = 0$ or $f'(c)$ does not exist.)

Step 2: Find points where f have a **vertical asymptote** or undefined. Answer: ____________

Step 3: Draw a number line, mark all points found in Steps 1 and 2, and find the sign of $f'(x)$ in each intervals between marked points.

Step 4: Write down the values of x for which f is increasing ($f'(x) > 0$) and those for which f is decreasing ($f'(x) < 0$).

▸ First derivative test for maxima and minima

Definitions: Let c be a point in the domain of a function $f(x)$.

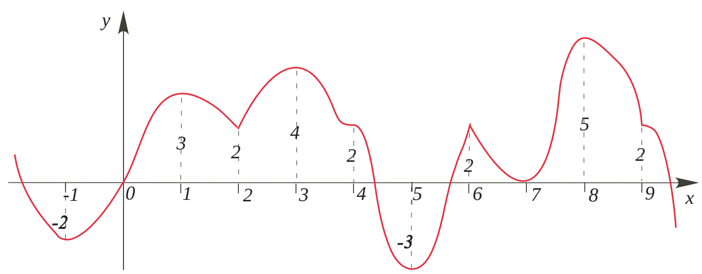
- $f(x)$ has a **local minimum** (or relative minimum) at c if ____________ for all x in an interval around c.
- $f(x)$ has a **local maximum** (or relative maximum) at c if ____________ for all x in an interval around c.

local extrema

- $f(x)$ has a **global minimum** (or absolute minimum) at c if $f(c) \leq f(x)$ for all x in the domain of f.
- $f(x)$ has a **global maximum** (or absolute maximum) at c if $f(c) \geq f(x)$ for all x in the domain of f.

global extrema

Example 2 Consider the following graph and locate all local and global extrema.



local minima: ______

local maxima: ______

global minima: ______

global maxima: ______

Fact: Critical points are the only *candidates* for extrema. **But** you may have a critical point that's not an extremum.

> **The first derivative test for maxima and minima**
> If $f(x)$ has a critical point at c, then
> - there is a local maximum at $x = c$ if $f'(x)$ changes its sign from positive to negative, and
> - there is a local minimum at $x = c$ if $f'(x)$ changes its sign from negative to positive.

Example 3 Find all critical points of the given function and use the derivative to determine where the function is increasing, where it is decreasing, and where it has a local maximum and minimum, if any.

(a) $f(x) = x^2 e^x$

(b) $f(x)$ is such that the graph of the **derivative** of $f(x)$ is given below.