Name ______________________________ Date ___________________

Math 10250 Activity 22: First Derivative Tests (4.1 continued)

GOAL: To use information given by $f'(x)$ to find where $f(x)$ is increasing and decreasing, and to locate maxima and minima.

First, let's review what we learned last time.

▸ The derivative test for increasing and decreasing functions

Method for finding where a function f is increasing/decreasing

1. Find all **critical points** of f. (That is, find all points in domain where $f'(x) = 0$ or $f'(x)$ does not exist.)
2. Find points where f has a **vertical asymptote** or is undefined.
3. Plot points in 1 and 2 on x-axis (making intervals).
4. Take one point a on each interval and compute $f'(a)$. The sign of $f'(a)$ is the sign of f' throughout that interval.
5. f is **increasing** on intervals where f' is ____________.
   f is **decreasing** on intervals where f' is ____________.

Example 1 Find all values of x for which $f(x) = \dfrac{1}{x^2 - x}$ is increasing or decreasing with the steps outlined below.

Step 1: Find all **critical points** of f. (That is all points c in the domain where $f'(c) = 0$ or $f'(c)$ does not exist.)

Step 2: Find points where f have a **vertical asymptote** or undefined. Answer: ____________

Step 3: Draw a number line, mark all points found in Steps 1 and 2, and find the sign of $f'(x)$ in each intervals between marked points.

Step 4: Write down the values of x for which f is increasing ($f'(x) > 0$) and those for which f is decreasing ($f'(x) < 0$).

The first derivative test for maxima and minima

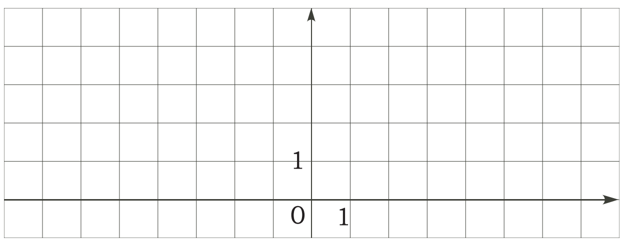
If $f(x)$ has a critical point at c, then

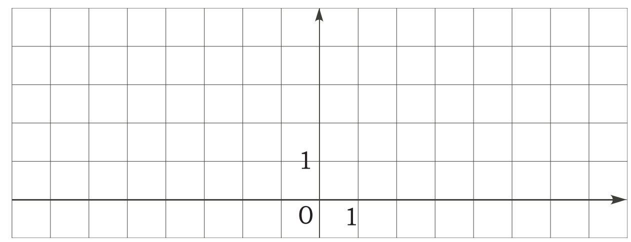
- there is a local maximum at $x = c$ if $f'(x)$ changes its sign from ________ to ________, and
- there is a local minimum at $x = c$ if $f'(x)$ changes its sign from ________ to ________.

Example 2 In Example 1, where does $f(x)$ have a local maximum or local minimum, if any?

Example 3 Sketch the graphs of **two different** functions sharing the same properties below. The graphs should have at least one feature that is markedly different.

- $f'(x) < 0$ on $(-\infty, 0)$ or $(2, \infty)$.
- $f'(x) > 0$ on $(0, 2)$.
- $f(0) = 0$ and $f(2) = 4$.
- $f'(0) = 0$ but $f'(2)$ does not exist.
- $\lim_{x \to +\infty} f(x) = 2 = \lim_{x \to -\infty} f(x)$.





▶ Global Maximum and Global Minimum

Q1: How can we determine the global maximum or global minimum of a given function?

A1: One way is to study how the function increases and decreases.

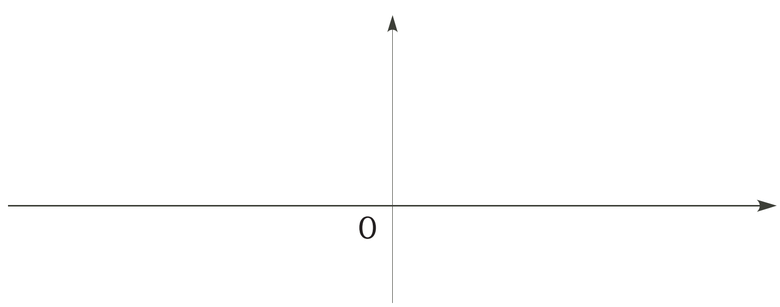
Example 4 Find the local and global extrema, if any, of $f(x) = x^2 e^{-x}$ for $-\infty < x < \infty$.

Step 1: Find all **critical points** of f.

Step 2: Find points where f have a **vertical asymptote** or undefined. Answer: ____________

Step 3: Find the values of $f(x)$ at all critical points, and behavior of $f(x)$ at $\pm\infty$.

Step 4: Give a rough sketch of the graph of $f(x)$ indicating clearly where f is increasing and decreasing.



Step 5: Read off all global maxima and global minima from the sketch above. If there are none state so.