Name ______________________________ Date ___________________

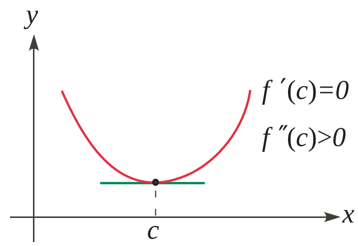
Math 10250 Activity 24: Second Derivative Tests (continued)

GOAL: To use information given by $f''(x)$ to locate local maxima and minima.

▶ The second derivative test for local maxima and minima

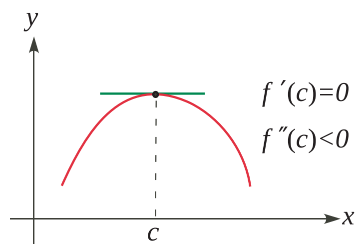
Q1: If $f'(c) = 0$ and $f''(c) > 0$, what can we conclude about c?

A1:
$f'(c) = 0 \quad \Rightarrow \quad$ Slope is zero at c.
$f''(c) > 0 \quad \Rightarrow \quad$ Graph is __________________ near c.
Therefore, c is a local ____________.



Q2: If $f'(c) = 0$ and $f''(c) < 0$, what can we conclude about c?

A2:
$f'(c) = 0 \quad \Rightarrow \quad$ Slope is zero at c.
$f''(c) < 0 \quad \Rightarrow \quad$ Graph is __________________ near c.
Therefore, c is a local ____________.



This gives us another test for local extrema.

Second derivative test
Suppose $f'(x)$ and $f''(x)$ exist around c and $f'(c) = 0$.
- If $\boldsymbol{f''(c) > 0}$ then there is a __________________ at $x = c$.
- If $\boldsymbol{f''(c) < 0}$ then there is a __________________ at $x = c$.
- If $\boldsymbol{f''(c) = 0}$ then the test is __________________.

Method for finding extrema of f using the second derivative test
1. Compute f' and find all critical points of f. (That is, find all points in domain where $f'(x) = 0$ or $f'(x)$ does not exist.)
2. Compute f'' and use second derivative test (on solutions to $f'(x) = 0$) to find local minima and maxima.

Example 1 Use Second derivative test to determine all local maximum and minumum points of $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 7$.

Example 2 Find the critical points of the $f(x) = x^5 - \frac{5}{4}x^4$ and determine whether each is a local minimum or maximum. Use the second derivative test wherever possible.

Example 3 A paint manufacturer needs to construct a cylindrical can that holds 250π cm^3 (≈ 785 cm^3) of its product. To reduce its cost for the can, the manufacturer needs to construct one with minimal surface area. Could you help the manufacturer to decide on the dimensions of the can? Note that the can is a cylinder closed on both ends. ($r = 5$ cm, $h = 10$ cm)

Example 4 Using the information for the **continuous** function $f(x)$ below, find all critical points of $f(x)$ and classify them using (i) the first derivative test, and (ii) the second derivative test if possible. State also all inflection points.

- $f'(x) < 0$ for $(-\infty, -2) \cup (-1, 0) \cup (0, 1)$.
- $f'(x) > 0$ for $(-2, -1) \cup (1, +\infty)$.
- $f'(-2) = 0 = f'(0)$; $f'(-1)$ and $f'(1)$ do not exist.
- $f''(0) = 0 = f''(2)$; $f''(-1)$ and $f''(1)$ do not exist.
- $f(x)$ is concave up for $(-\infty, -1) \cup (-1, 0) \cup (1, 2)$.
- $f(x)$ is concave down for $(0, 1) \cup (2, +\infty)$.