Name _____ Date _____

Math 10250 Activity 27: Optimization (Sect. 4.4 cont.) and Applied Optimization Problems (Sect. 4.5)

GOAL: To find maximum and minimum of a function over an interval with one or both endpoints.

▶ **Case 1: Optimizing $f(x)$ on a closed interval** (Done in last class)

Example 1 Find the global maximum and minimum of the function $f(x) = xe^{-x/2}$ for $[1, 4]$. Give a sketch of the graph of $f(x)$ clearly indicating where the global maximum and minimum are.

▶ **Case 2: Optimizing $f(x)$ on an interval with one or both endpoints excluded (i.e., on $(a, b], (-\infty, b], [a, \infty), (-\infty, \infty), \ldots$) - Global maximum and minimum may or may not exist.**

Example 2 Using the steps below, find the global maximum and minimum of the function $f(x) = xe^{-x/2}$ on $[1, \infty)$.

Step 1: Find all critical points in the domain of $f(x)$ and the values of $f(x)$ there. Classify them using first derivative test.

Step 2: Find all asymptotes of $f(x)$ in its domain and determine its asymptotic behavior.

Step 3: Find the values of $f(x)$ at the end-points (if any) of its domain. _____

Step 4: Give a rough sketch of the graph of $f(x)$ clearly indicating where the global maximum and minimum are. Stating the global maximum and minimum of $f(x)$ on $[1, \infty)$ if any.

Q1: How does Example 2 contrast with Example 1?

A1:

Example 3 Find the global maximum and minimum of the $f(x) = x^4 - 8x^2$ on $(-\infty, 1)$.

Step 1: Find all critical points in the domain of $f(x)$ and the values of $f(x)$ there. Classify them using first derivative test.

Step 2: Find all asymptotes of $f(x)$ in its domain and determine its asymptotic behavior.

Step 3: Find the values of $f(x)$ at the end-points (if any) of its domain. _____

Step 4: Give a rough sketch of the graph of $f(x)$ clearly indicating where the global maximum and minimum are. Stating the global maximum and minimum of $f(x)$ on $(-\infty, 1)$ if any.

NEXT GOAL: To use our optimization methods to solve word problems.

Example 4 A restaurant owner discovers from the sales of an octopus dish that its average number of order q each night is given by $p = \dfrac{72}{q+2}$ where p is the price in dollars of an order of the dish. Suppose that each appetizer costs the restaurant \$4 to make. Help the owner of the restaurant with the following calculation:

(a) Write down the revenue function. _____

(b) What is the largest amount of revenue the restaurant can make from the appetizer?

(c) What price should the owner charge in order to maximize profit from the appetizer?