

Math 10250 Activity 29: The Indefinite Integral (Section 5.1)

GOAL: If we are given the derivative $f'(x)$, we want to be able to find the function $f(x)$.

► **Antiderivatives** (Reversing differentiation)

Definition: $F'(x) = f(x)$ means that $F(x)$ is an **antiderivative** of $f(x)$.

Example 1 Find all antiderivatives of the indicated function $f(x)$. That is find all $F(x)$ so that when we take its derivative, we get $f(x)$. In each case, sketch three of them on the same set of axes.

$$f(x) = 0$$

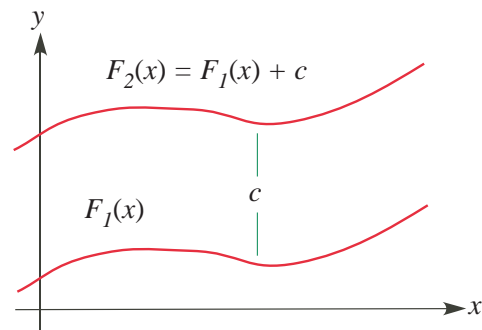
$$f(x) = x$$

From Example 1, we see that

Theorem If $F_1(x)$ and $F_2(x)$ are antiderivatives of the same function throughout an interval, then they differ by a constant c over that interval; that is, for $a < x < b$

$$F_1'(x) = F_2'(x) \Rightarrow$$

for some number c .



Q1: How do we denote all antiderivatives of $f(x)$?

A1: If $F(x)$ is an antiderivative of $f(x)$, that is, $F'(x) = f(x)$. Then we may write

$$\int f(x)dx = \underline{\hspace{4cm}}$$

We call $\int f(x)dx$ the **indefinite integral**.

Example 2 Let $f(x) = (5x - 1)^3$ and $F(x) = A(5x - 1)^4$.

a. Find the value of the constant A that makes $F(x)$ an antiderivative of $f(x)$.

b. Write your result in Part (a) in terms of indefinite integral.

Example 3 Referring to Example 2, find the indefinite integral of $f(x) = x$.

Example 4 If $k \neq 0$, compute $\frac{d}{dx}(e^{kx})$ then write down $\int e^{kx} dx$. (Use the fact: $(c \cdot f(x))' = c \cdot f'(x)$)

Example 5 If $k \neq -1$, compute $\frac{d}{dx}(x^{k+1})$ then write down $\int x^k dx$.

► **Basic indefinite integral formulas**

• For any constant k : $\int k dx \stackrel{?}{=} \boxed{}$. For Example: $\int 100 dx \stackrel{?}{=} $

• Power Rule when $k \neq -1$: $\int x^k dx \stackrel{?}{=} \boxed{}$. For Example: $\int x^9 dx \stackrel{?}{=} $

• Power Rule when $k = -1$: $\int \frac{1}{x} dx = \boxed{}$.

• Exponential Rule: $\int e^{kx} dx = \boxed{}$, $k \neq 0$ For Example: $\int e^{0.1x} dx \stackrel{?}{=} $

• Constant Multiple Rule: $\int kf(x)dx = k \int f(x)dx$, any k For Example: $\int \frac{8}{x} dx \stackrel{?}{=} $

• Sum Rule: $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$.

Example 6 Find each of the following indefinite integral. Check your answer by differentiation.

a. $\int \left(x^7 - 2x^{-4} + \frac{3}{x} + e^{2x} \right) dx$

b. $\int \frac{3x - 10x^2 + \sqrt{x}}{x^3} dx$

Example 7 Given that $\int f(x)dx = F(x) + C$ and $G'(x) = g(x)$. Find each of the following indefinite integral in terms of $F(x)$, $G(x)$, and other known functions whenever possible. If not possible state so.

a. $\int [2f(x) + 3x] dx$

c. $\int \frac{5 - 3x \cdot g(x)}{x} dx$

b. $\int f(x) \cdot g(x) dx$

d. $\int \frac{f(x) + 3}{x} dx$