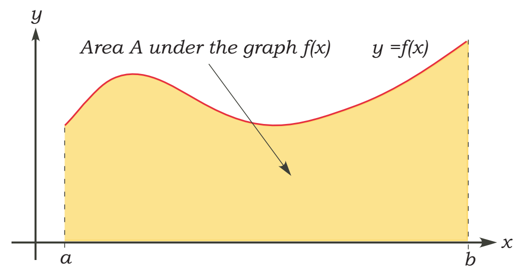
Name ______________________________ Date ___________________

Math 10250 Activity 33: Area and the Definite Integral (Section 5.4)

Goal: To compute area of **curved** regions in the plane.

• First consider the region under the graph of a non-negative function $f(x)$ over its domain $[a, b]$:

Q: How do you compute the area of the region A?



A: In five steps:

(1) Divide the interval $[a, b]$ into n equal subinterval.

(2) Choose a point in each subinterval.

(3) Compute the area of the rectangle corresponding to each piece.

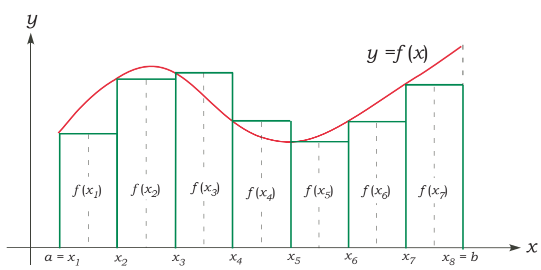
(4) Estimate the area of A by adding the areas of all rectangles.

(5) Get the exact area by taking larger and larger n (smaller and smaller subintervals).

Let's look at each of the above steps in detail.

(1) Divide $[a, b]$ into n subintervals of equal width Δx and choose a point x_i in each subinterval.

(Usually this point is chosen to be either the left endpoint, the right endpoint, or the midpoint of the subinterval.)



$\Delta x =$ ______	← **mesh** of the partition

(2) Construct a rectangle over each subinterval with height $f(x_i)$ and compute the area of each rectangle. (Let's use left-hand endpoints of the segments)

Area of first rectangle = height · base = ________________.

Area of second rectangle = height · base = ________________.

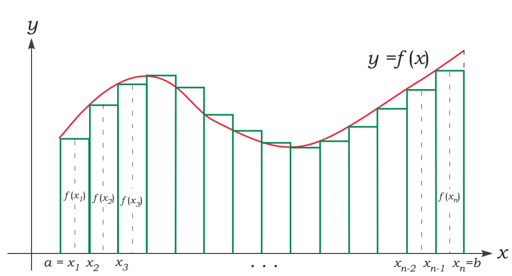
⋮

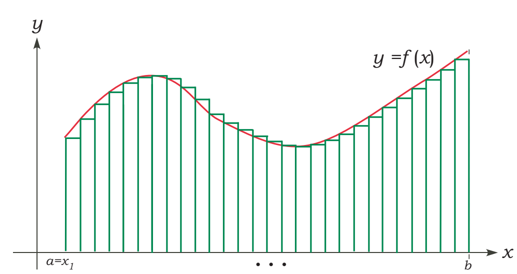
Area of nth rectangle = height · base = ________________.

(3) Estimate the area of A by adding the areas in (2).

area of $A \approx$ $\qquad\qquad\qquad\qquad$ $= S_n(f)$	← **Riemann Sum**

(4) The approximation above gets more accurate as the rectangles get smaller.





(5) So we can get the exact area of A by letting ___________. Therefore,

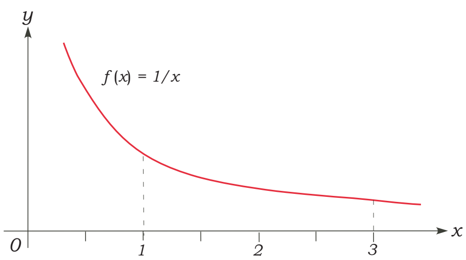
$$\text{area of } A = \lim_{n\to\infty}[f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x] = \lim_{n\to\infty} S_n(f)$$

Example 1 Estimate the area under the graph of $y = 1/x$, $1 \le x \le 3$, by partitioning the interval $[1, 3]$ into 4 equal segments and computing the Riemann sum

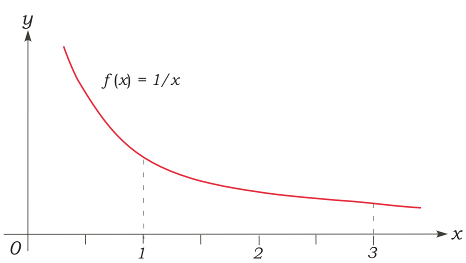
$$S_4(f) = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x,$$

where the points x_1, x_2, x_3, and x_4 are chosen to be:

(a) left-hand endpoint of the segments,



(b) midpoint of the segments.



▶ The definite integral: nonnegative case

The limit in Step (5) on the previous page is so special that we give it a name and symbol:

$$= \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$$ ← **Definition of the Definite Integral**

↑

area under the graph of $f(x)$ for $a \le x \le b$ if $f(x)$ is nonnegative

If this limit exists, we call it the **Definite Integral of $f(x)$ over the interval** $a \le x \le b$.

Example 2 Find $\int_0^1 4x\, dx$ using geometry.