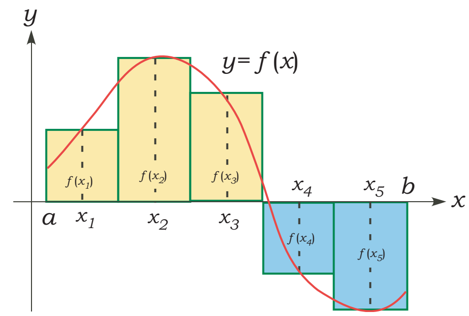
Name ________________________________ Date ____________________

Math 10250 Activity 34: Area and the Definite Integral (Sections 5.4 & 5.5)

GOAL: To compute the definite integral of a function that takes both positive and negative values.

Q1: How do we define the integral of a function that can take both positive and negative values?

A1: In four steps:



(1) Divide the interval $[a, b]$ into n subintervals.

(2) Choose a point in each subinterval.

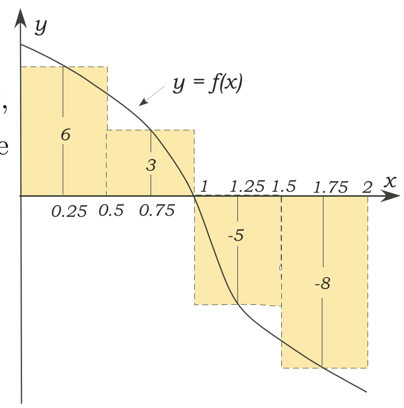
(3) Compute the corresponding Riemann sum.

$S_n(f) =$

← **Riemann Sum**

(4) By letting n go to infinity obtain:

$$\int_a^b f(x)dx = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x] \,.$$ ← **Definite Integral**



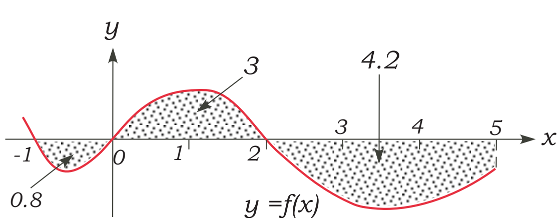
Example 1 For the function $f(x)$ whose graph is displayed in the Figure below, estimate $\int_0^2 f(x)dx$ by using the Riemann sum corresponding to $\Delta x = 0.5$ and the midpoints.

Example 2 Estimate the integral $\int_{-1}^{1} x^3 e^{-x^2} dx$ using 4 strips with left-hand endpoints.

• The relation between integral and area is:

$$\int_a^b f(x)\,dx = \text{(area of region lying \_\_\_\_\_\_\_\_ the } x\text{-axis)} - \text{(area of region lying \_\_\_\_\_\_\_\_ the } x\text{-axis)}.$$

Example 3 The graph of $f(x)$ for $-1 \le x \le 5$, is shown in the figure below. The size of each enclosed area are as indicated.



$$\int_{-1}^{0} f(x)\,dx \overset{?}{=} \qquad \int_{0}^{2} f(x)\,dx \overset{?}{=} \qquad \text{and} \int_{2}^{5} f(x)\,dx \overset{?}{=}$$

(a) Find the area of the region **enclosed** by the graph of $f(x)$, $-1 \le x \le 5$, and the x-axis.

(b) $\int_{-1}^{5} f(x)dx \overset{?}{=}$

Q2: What are the basic properties of definite integral?
A2:

▶ The Fundamental Theorem of Calculus (Section 5.5)

Q1: What is the connection between $\int_a^b f(x)\,dx$ and $\int f(x)\,dx$?

↑ definite integral ↑ indefinite integral

A2:

Fundamental Theorem of Calculus		
IF	(1) $f(x)$ is continuous on $[a,b]$;	(2) $F(x)$ is an antiderivative of $f(x)$ i.e $F'(x) = f(x)$
THEN	$\int_a^b f(x)dx =$ ____________	i.e. $\int_a^b F'(x)dx =$ ____________

Example 4 Compute the following definite integrals:

(a) $\int_1^2 (x^2+3)\,dx$

(b) $\int_{-2}^{-1} (e^{2x} + \frac{2}{x})\,dx$

Example 5 Sketch the graph of $f(x) = 2e^x$ from $a = -1$ to $b = 2$ and use the fundamental theorem of calculus to find the area under the graph.

$\int_{-1}^{2} 2e^x\,dx =$

▶ Physical interpretations of the Fundamental Theorem of Calculus

** **Total change** of a certain quantity is expressed as the **definite integral** of its rate of change.**

• **From velocity** v **to displacement** s:

Displacement between times a and $b = s(b) - s(a) = \int_a^b$ ______ dt.

↑ change of position between times a and b

Example 6 An object is falling vertically downward, and its velocity (in feet per second) is given by $v = -32t - 20$. Write a definite integral that gives the change in height in the first 3 seconds.

Similary, the following are true.

• **From acceleration** a **to velocity** v:

Change in velocity between times a and $b = v(b) - v(a) = \int_a^b$ ______ dt.

• **From rate of growth** $r(t)$ **to total growth** $g(t)$:

Total growth between times a and $b = g(b) - g(a) = \int_a^b$ ______ dt.