Name ______________________________ Date ___________________

Math 10250 Activity 35: More on Definite Integrals (Section 5.5 Continue & 5.6)

Goal: Introduce area function. Applying method of substitution and integration by parts with Fundamental Theorem of Calculus.

▶ From marginal function to total function

- The additional profit resulting in increasing production from a units to b units is given by

Total change in profit $\overset{?}{=}$ $\qquad\qquad$ $\overset{?}{=}$ $\qquad\qquad$ $= \int_a^b MP(x)\, dx.$

- The extra revenue resulting from increasing production from a units to b units is given by

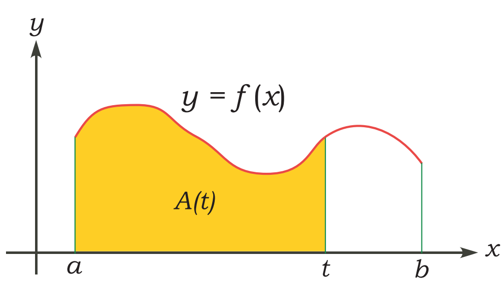
Total change in revenue $\overset{?}{=}$ $\qquad\qquad$ $\overset{?}{=}$ $\qquad\qquad$ $\overset{?}{=}$ $\qquad\qquad$.

Example 1 Suppose the marginal cost involved in producing x units of a certain product is given by the function

$$MC(x) = 2x + 1000 \text{ when } x \geq 50.$$

Determine the increase in cost if production is increased from 50 to 80.

▶ The area as an antiderivative



Let $A(t) = \int_a^t f(x)\, dx$ for $a \leq t \leq b$.

If $F(t)$ is an antiderivative of $f(t)$, what is the relation between $A(t)$ and $F(t)$? (Hint: Fundamental Theorem of Calculus)

Conclusion: $A(t)$ is *also* an antiderivative of $f(t)$. i.e.,

Theorem 5.5.2
IF $f(x)$ is continuous on $[a, b]$ THEN $\dfrac{d}{dt}\int_a^t f(x)dx \overset{?}{=}$

Example 2 $\dfrac{d}{dt}\int_1^t (1 + \ln x)^2\, dx \overset{?}{=}$.

▶ Substitution in definite integrals

$$\boxed{\int_a^b f(g(x))g'(x)\ dx \stackrel{u=g(x)}{=}}$$

Example 3

(a) $\int_4^5 x\sqrt{x^2-16}\ dx \stackrel{?}{=}$

(b) $\int_0^1 xe^{x^2}\ dx \stackrel{?}{=}$

▶ Integration by parts in definite integrals

$$\boxed{\int_a^b u\ dv =}$$

Example 4

(a) $\int_0^1 te^{t/2}\ dt \stackrel{?}{=}$

(b) $\int_1^2 x\ln(x)\ dx \stackrel{?}{=}$