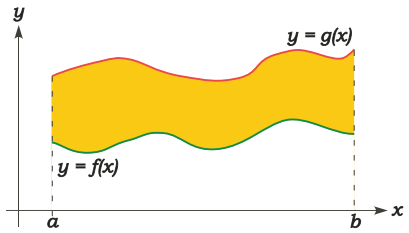
Name _____ Date _____

Math 10250 Activity 36: More on Areas and Estimating Definite Integrals
(Section 5.6 continue & 5.7)

GOAL: To compute definite integrals and areas between the graphs of two functions. Introduce more numerical methods for integration.

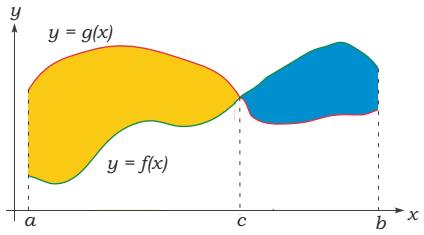
▶ The area between two curves

Consider the following region:



Area between f and $g = \int_a^b$ _____ , if $f(x) \le g(x)$.

Now consider:



Area between f and g
$= \int_a^c [g(x) - f(x)]\, dx + \int_c^b [f(x) - g(x)]\, dx.$

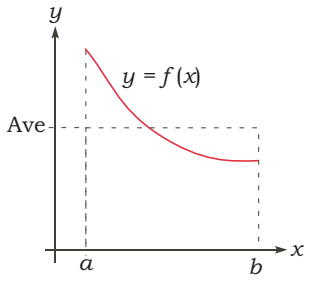
Example 1 Find the intersection points of $f(x) = 1 - x^2$ and $g(x) = x + 1$. Then find the area between the graphs over the interval $-1 \le x \le 1$.

▶ Average values of continuous quantities

Q1: What is the average value of $3, 5$ and 7? **Answer:** _____

Q2: What is the average value of $f(x)$ on $[a, b]$?

A2: Average value of f over $[a, b] = \frac{\int_a^b}{\text{_____}}$



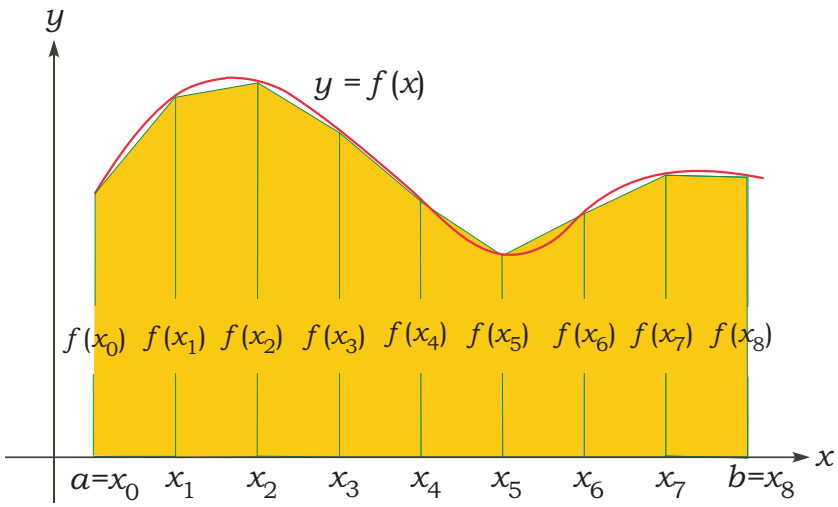
Example 2 Find the average value of $f(x) = x^5$ over the interval $[-1, 2]$.

Example 3 Estimate $\int_0^1 e^{\sqrt{x}}\, dx$

(a) using the mid-point with $n = 4$

(b) using trapezoidal rule with $n = 4$.

► **The trapezoidal rule:** To estimate $\int_a^b f(x)\, dx$, we can use trapeziods instead of rectangles. Recall that the area of a trapezoid $= \frac{1}{2}$(sum of the two parallel sides)·(height). Apply this method for the function below.



$$\int_a^b f(x)\, dx \approx [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)] \cdot \frac{\Delta x}{2} \leftarrow \textbf{trapezoidal rule}$$