Name _____ Date _____

Math 10250 Review for Exam 1

1. (a) Determine the natural domain of $f(x) = \dfrac{2-x}{x-1}$, find also its inverse $g(x)$. Ans. $x \neq 1$; $g(x) = \frac{2-x}{x+1}$

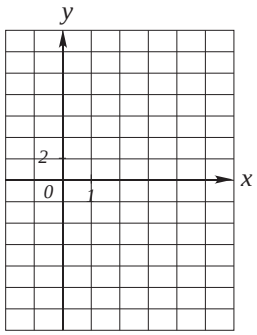
   (b) What is the natural domain of $f(x) = \sqrt{3-2x}$? Ans. $x \leq 3/2$

2. A brand of sunglasses selling for \$50 each has a demand of 1,500 units. However, when the price is **increased** by \$5, its demand is **decreased** by 100 units. Find its demand assuming that is a linear function. Ans. $q = D(p) = -20p + 2,500$

3. Complete the square for each quadratic and then sketch its graph.

   (i) $f(x) = -3x^2 + 12x$

   (ii) $f(x) = 2x^2 - 12x + 10$.



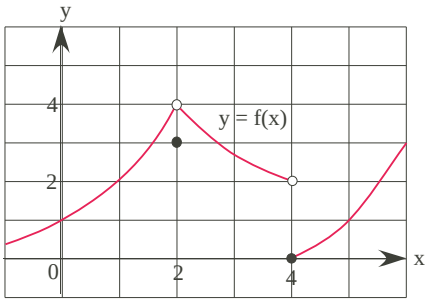
Ans. (i) $f(x) = -3(x-2)^2 + 12$; (ii) $f(x) = 2(x-3)^2 - 8$

4. When the price p of a particular computer is \$2,000 then the demand x is 50,000 units per week. However, when the price drops by \$500 then the demand rises by 25,000 units. On the cost side, the company making these computers has \$40,000,000 fixed cost and \$600 expenses per unit. Assuming that the demand is linear , find the profit function P in terms of x and its maximum value.

Ans. $P = -0.02x^2 + 2{,}400x - 40{,}000{,}000 = -0.02(x - 60{,}000)^2 + 32{,}000{,}000$

5. (a) $\lim\limits_{h\to 0} \dfrac{5(1+h)^2 - 5}{h} \stackrel{?}{=}$ (b) $\lim\limits_{h\to 0} \dfrac{\frac{1}{2+h} - \frac{1}{2}}{h} \stackrel{?}{=}$ Ans. (a) 10; (b) $-\frac{1}{4}$

6. The graph of the function $f(x)$ is given in the next Figure. Which of the following statements is **NOT** true?
   (**a**) $\lim\limits_{x\to 2} f(x) = 4$
   (**b**) $\lim\limits_{x\to 4^-} f(x) = 2$ and $\lim\limits_{x\to 4^+} f(x) = 0$
   (**c**) $f(x)$ has limit at $x = 4$.
   (**d**) $f(x)$ is continuous except at the points $x = 2, 4$.
   (**e**) $\lim\limits_{x\to 0} f(x) = 1$.



Ans. c

7. If $x \neq 2$ then $f(x) = \dfrac{x^2 + 2x - 8}{x - 2}$. Define $f(2)$ so that $f(x)$ is a continuous function. Ans. $f(2) = 6$

8. In which of the following intervals you can be sure that the function $f(x) = x^4 + 2x^3 - 3x^2 - 2x + 3$ takes the value 2? (i.e the equation $f(x) = 2$ has a solution.) $[-3,-2], [-2,-1], [-1,0], [0,1], [1,2], [2,3]$

Ans. $[-3,-2], [-1.0], [0,1], [1,2]$

9. For each function below, find vertical asymptote(s), horizontal asymptote(s), y-intercept, its zero(s), and then sketch its graph. Ans. (a) v.a: $x = \pm 1$; h.a: $y = 1$; zeros: $x = \pm 4$, y-intercept: 16; (b) v.a: $x = -1$; h.a: $y = 0$; zeroes: None, y-intercept: None

   (a) $f(x) = \dfrac{x^2 - 16}{x^2 - 1}$ (b) $f(x) = \dfrac{x - 4}{x^2 - 3x - 4}$

10. Suppose that you put \$100 in an account paying 2% annual interest, compounded daily. How much will you have at the end of 1 day? 2 days? and 3 days? Ans. $100\left(1 + \frac{0.02}{365}\right)$; $100\left(1 + \frac{0.02}{365}\right)^2$; $100\left(1 + \frac{0.02}{365}\right)^3$

11. Suppose that you have an account paying interest, compounded weekly, has balance given by $B(t) = 8000(1.0004)^{52t}$. What is its principal and annual interest rate? Ans. $P = 8000$; $r = 2.08\%$

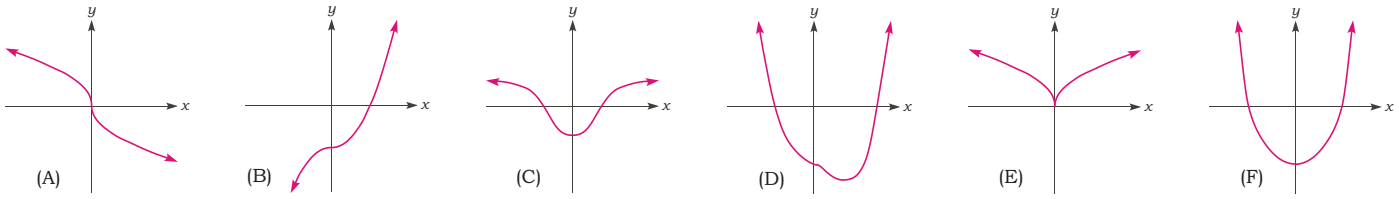
12. \$4,000 is deposited into an account paying $q\%$ interest, compounded **annually**. If the account doubles after 10 years, what is q? Ans. $q = 7.18$

13. A population of bacteria on a growing medium is initially 10 million. Three hours later the number of bacteria is numbered at 15 million. Write down a formula for the population $P(t)$ at time t in hours if the population is growing exponentially. Ans. $P(t) = 10(1.145)^t$

14. Match the following functions with the given graphs without using your calculator:

$f_1(x) = -x^{1/3}$ $\quad f_2(x) = x^{2/3}$ $\quad f_3(x) = x^4 - x - 5$

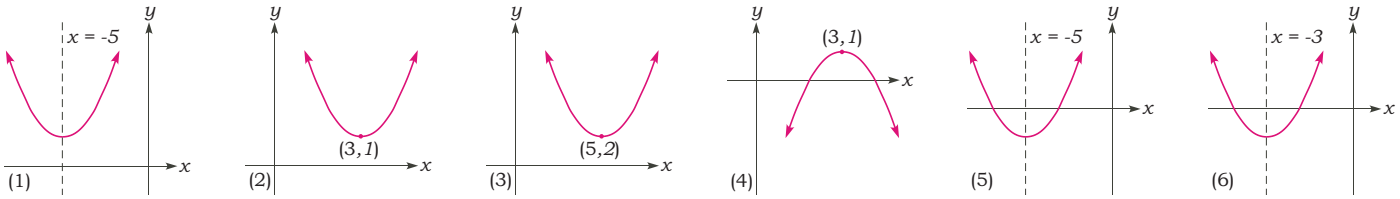
$f_4(x) = \dfrac{5x^4 - 25}{x^2 + 5}$ $\quad f_5(x) = \dfrac{5x^3 - 25}{x^2 + 5}$ $\quad f_6(x) = \dfrac{5x^2 - 25}{x^2 + 5}$



15. Match the graphs to the given quadratic functions. Some graphs are redundant.

$f_1(x) = (x - 5)^2 + 2$ $\quad f_2(x) = a(x - 3)^2 + 1 \quad (a < 0)$

$f_3(x) = b(x + 3)^2 - 1 \quad (b > 0)$ $\quad f_4(x) = (x + 5)^2 + 2$



16. A private health club has determined that the number of members depends on the price of a membership, and they are related by an equation of the form $q = 3000 - 20p$, where q is the number of members and p is the annual price of a membership. The club has a fixed costs of \$20,000 per year plus an average annual cost of \$40 per member.

    (a) Write the club's revenue R as a function of the price p. Ans. $R = 3000p - 2p^2$

    (b) Write the club's profit P as a function of the price p. Ans. $P = -20p^2 + 3800p - 140000$

    (c) What membership price should the club set to maximize its profit? Ans. \$95

    (d) Find the break-even point. Interpret your answer. Ans. \$50 and \$140

17. Find the equilibrium price p_e and equilibrium quantity q_e for each pair of demand and supply functions. Make a sketch of the graphs marking the coordinates of intersection point.

    (a) $D(q) = 0.005(q - 100)^2$ and $S(q) = 0.1q + 2$ for $0 \le q \le 100$ Ans. $p_e = 8$, $q_e = 60$

    (b) $D(p) = \dfrac{8}{p + 1}$ and $S(p) = \dfrac{1}{3}p + 1$ Ans. $p_e = 3$, $q_e = 2$