

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc.
I
Fall 2008

Name:_____

Instructor:_____

Exam I

September 18, 2008

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature:_____

You must record here your answers to the multiple choice problems.

Place an \times through your answer to each problem.

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. _____

11. _____

12. _____

13. _____

14. _____

Tot. _____

Multiple Choice

1. (5 pts.) In 2007 U.S. cumulative wind energy capacity reached 16.8 gigawatts (GW) and was growing at the rate of about 5.2 GW per year¹. Assuming that after 2007 wind energy w , in GW, is modeled by a linear function of time, in years, write an equation for w in terms of t . (Think of 2007 as zero time.)

(a) $w = 16.8 - 5.2t$

(b) $w = 22t$

(c) $w = 5.2t$

(d) $w = 5.2 + 16.8t$

(e) $w = 16.8 + 5.2t$

2. (5 pts.) In 2007 there were about 17 GW wind power generated in the U.S. If after this time wind power is modeled by a linear function of time, find the rate at which wind power must be generated so that it reaches the value of 304 GW in 2030.²

(a) 291/23 GW/year (b) 287/23 GW/year (c) 290/23 GW/year

(d) 288/23 GW/year (e) 289/23 GW/year

¹<http://www.doe.gov/news/6253.htm>

²This year (2008) the U.S Department of Energy (DOE) released a first-of-its kind report (see: <http://www.doe.gov/news/6253.htm>) that examines the technical feasibility of harnessing wind power to provide up to 20 percent of the nations total electricity needs by 2030, which is estimated to be about 304 GW.

3. (5 pts.) Assume that the population p of gray whales in the North Pacific is modeled by the function

$$p(t) = 20,500 + \frac{10,000}{t+2},$$

where t is the number of years after 2000. Which of the following statements is **FALSE**?

- (a) In 2018 the gray whales population will be 21,000.
- (b) In the distant future the gray whales population will be approximately 20,500.
- (c) In the distant future gray whales will disappear.
- (d) $p(t)$ is a decreasing function of t for all $t \geq 0$.
- (e) In 2000 the gray whales population was 25,500.

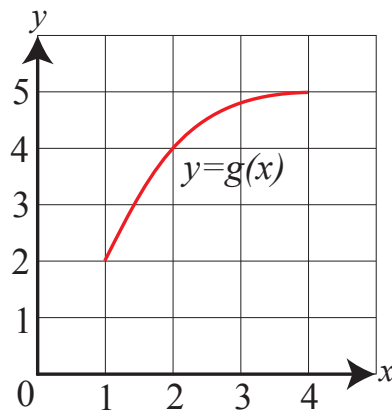
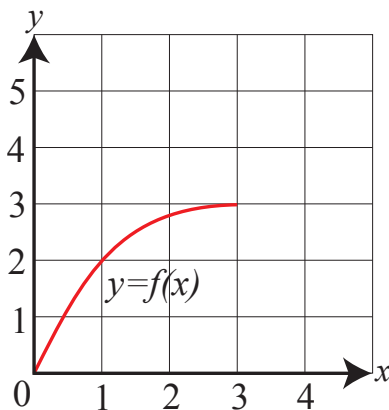
4. (5 pts.) Compute the following limit: $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$

- (a) $1/4$
- (b) $1/2$
- (c) Does not exist.
- (d) $1/5$
- (e) $1/3$

5. (5 pts.) Let $f(x) = \frac{10x^4 + 7}{2x^4 + 1}$. Which of the following statements is **FALSE**?

- (a) The function $f(x)$ is symmetric about the y -axis.
- (b) The function $f(x)$ is even.
- (c) The function $f(x)$ is symmetric about the origin.
- (d) The natural domain of $f(x)$ is the set of all real numbers.
- (e) The value of $f(x)$ approaches 5 as x gets unboundedly large.

6. (5 pts.) The graphs of $f(x)$ and $g(x)$ are given below.



Which of the following expression describes the relationship between $f(x)$ and $g(x)$?

- (a) $g(x) = f(x + 1) + 2$
- (b) $g(x) = f(x - 1) + 4$
- (c) $g(x) = f(x - 1) - 2$
- (d) $g(x) = f(x - 1) + 2$
- (e) $g(x) = f(x - 3) + 4$

7. (5 pts.) Sonja thinks that her weight tends to oscillate around 145 pounds. To verify this she has recorded her weight w at the beginning of each month for the last six months, thus constructing the following table:

t	1	2	3	4	5	6
$w(t)$	148	147	144	142	143	147

Assuming that her weight $w(t)$ is a **continuous** function of t , in which of the following time intervals can you be sure that Sonja weight $w(t)$ attained the value of 145 pounds?

- (a) $[1, 2]$ and $[2, 3]$ only.
- (b) $[3, 4]$ and $[4, 5]$ only.
- (c) $[2, 3]$ and $[3, 4]$ only.
- (d) $[2, 3]$ and $[5, 6]$ only.
- (e) $[1, 2]$ and $[5, 6]$ only.

8. (5 pts.) A telephone company charges the rate of 12 cents per minute between 8 A.M. and 4 P.M., and 6 cents per minute for the rest of the day. Which of the following statements is **not** true for the rate function $r(t)$, when $0 < t < 24$?

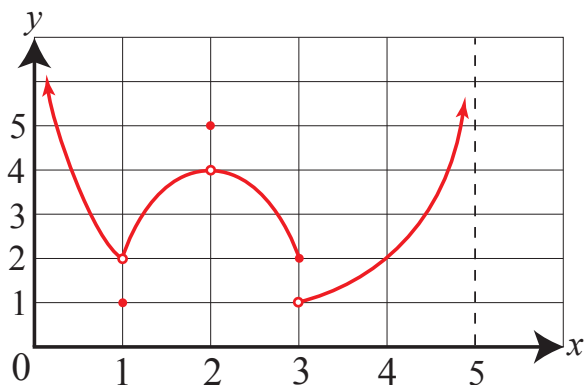
- (a) $r(t)$ has no limit at $t = 8, 16$.
- (b) The right-hand limit of $r(t)$ at $t = 8$ is 12.
- (c) The left-hand limit of $r(t)$ at $t = 16$ is 6.
- (d) The left-hand limit of $r(t)$ at $t = 8$ is 6.
- (e) $r(t)$ is not continuous at $t = 8, 16$.

9. (5 pts.) You have just deposited \$1000 in your bank account at the ND Credit Union that pays annual interest 5% compounded **daily**. Assuming that the year has **365 days**, find the balance of your account after 80 days.

- (a) $1000 \left(1 + \frac{1}{7300}\right)^{80}$ (b) $1000 \left(1 + \frac{1}{7500}\right)^{80}$ (c) $1000 \left(1 + \frac{1}{7400}\right)^{80}$
 (d) None of the above. (e) $1000 \left(1 + \frac{1}{7600}\right)^{80}$

10. (5 pts.) Let $f(x)$ be the function whose graph is shown below. Which of the following statements is **FALSE**?

- (a) $f(x)$ has a limit at $x = 1$.
 (b) $f(x)$ is not continuous at $x = 1$.
 (c) $f(x)$ has a limit at $x = 3$.
 (d) $\lim_{x \rightarrow 4} f(x)$ exists.
 (e) $\lim_{x \rightarrow 5^-} f(x) = +\infty$.



Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (14 pts.) The profit, in thousands of dollars, from the sales of a certain item is given by the formula

$$P(x) = -x^2 + 20x - 75,$$

where x is the number of hundreds of the item sold.

(i) (6 pts.) By completing the square, write $P(x)$ in the form $P(x) = a(x - h)^2 + k$. Show clearly all your steps.

(ii) (4 pts.) Write the maximum profit and the value of x when profit is maximum?

Maximum profit $\stackrel{?}{=}$ _____ when $x \stackrel{?}{=}$ _____.

(iii) (4 pts.) What are the **break-even** points (where profit is zero)?

12. (14 pts.)**Part A.** (6 pts.) Find the **equations** of **all** vertical and horizontal asymptotes of the following functions. If there is none, circle “NONE”.

(i) $f(x) = \frac{x^2 - 4}{x^2 - 7x + 10}$

Equations

Vertical: _____ NONE

Horizontal: _____ NONE

Part B. (6 pts.) (Independent from Part A.) Find the value of c such that the function $f(x)$ below is continuous at $x = 3$:

$$f(x) = \begin{cases} \frac{x^2 + 2x - 15}{x - 3}, & \text{if } x \neq 3 \\ c, & \text{if } x = 3, \end{cases}$$

Explain your work clearly using the limit definition of continuity.**Answer:** _____

13. (14 pts.) (A) Suppose that the demand curve for a certain commodity is modeled by

$$p = \frac{8,000}{q+4}$$

where p is the price and q is the quantity.

(i) (4 pts.) Compute the revenue function R .

(ii) (4 pts.) Compute $\lim_{q \rightarrow \infty} R(q)$.

(B) (4 pts.) (Independent of A.) Assume that the demand $D(p)$ and supply $S(p)$ for a certain item are given by

$$D(p) = -18p + 2,200 \quad \text{and} \quad S(p) = 12p - 800.$$

Find the equilibrium price p_e and equilibrium quantity q_e .

14. (14 pts.)

(A) (6 pts.) How much money should be put in an account paying 5% interest, compounded quarterly (four times a year), in order to have \$100,000 ten years from now?

(B) (Independent of A.) (6 pts.) A certain population is growing exponentially. In 2 years, the population grew from 1,000 to 9,000. Write a formula for the size of the population $P(t)$ as a function of time t in years. (Hint: Write $P(t) = P_0 \cdot b^t$)

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc.
I
Fall 2008

Name:_____

Instructor:_____

Exam I

September 18, 2008

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature:_____

You must record here your answers to the multiple choice problems.

Place an \times through your answer to each problem.

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (●) |
| 2. | (a) | (●) | (c) | (d) | (e) |
| 3. | (a) | (b) | (●) | (d) | (e) |
| 4. | (●) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (●) | (d) | (e) |
| 6. | (a) | (b) | (c) | (●) | (e) |
| 7. | (a) | (b) | (c) | (●) | (e) |
| 8. | (a) | (b) | (●) | (d) | (e) |
| 9. | (●) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (●) | (d) | (e) |

MC. _____

11. _____

12. _____

13. _____

14. _____

Tot. _____