Math 10250 Exam 1 Solutions – Fall 2008

1 At $t = 0$ wind energy production was $w = 16.8$ MW and was growing at the rate (slope) 5.2 MW/year. Using the point-slope formula we obtain the equation $w - 16.8 = 5.2(t - 0)$, or $w = 5.2t + 16.8$.

2 At $t = 0$ wind energy production was $w = 17$GW while at $t = 23$ it must reach the value $w = 304$GW. Therefore it should increase at the rate of (304-17)/(23-0)=287/23.

3 In the distant future the population of the whales is given by $\lim_{t\to\infty} p(t) = 20,500 + \lim_{t\to\infty} \dfrac{10000}{t+2} = 20,500$. Therefore, the whales will not disappear.

4 Since $\dfrac{\sqrt{4+h}-2}{h} = \dfrac{(\sqrt{4+h}-2)((\sqrt{4+h}+2)}{h(\sqrt{4+h}+2)} = \dfrac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)} = \dfrac{1}{\sqrt{4+h}+2}$, the given limit is equal to $\lim_{h\to 0} \dfrac{1}{\sqrt{4+h}+2} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$.

5 $f(x)$ is even and thus its graph is symmetric with respect to the y-axis. However $f(x)$ is not odd, i.e., $f(-x) \neq -f(x)$. Thus $f(x)$ is *not* symmetric with respect to the origin. Since $2x^4 + 1 \neq 0$ its natural domain is all real numbers. Also, $\lim_{x\to\infty} f(x) = 10/2 = 5$.

6 The graph of $g(x)$ is obtained by translating the graph of $f(x)$ horizontally by 1 unit and vertically by 2. Therefore $g(x) = f(x-1) + 2$.

7 Since Sonja's weight is a continuous function her weight assumes the value 145 in the time intervals $[2, 3]$ and $[5, 6]$ for certain, since only there it moves from above(below)145 to below(above)145.

8 The function $r(t)$ has no limit at $t = 8$ and 16 since there the right-hand and the left-hand limits are different. Thus the function is not continuous at $t = 8$ and 16. Note that the left-hand limit is $r(t)$ at $t = 16$ is 12, not 6.

9 A day later the balance of the account will be $1000 + 1000 \cdot \frac{0.05}{365} = 1000\left(1 + \frac{0.05}{365}\right)$. Two days later the balance will be $1000\left(1 + \frac{0.05}{365}\right)^2$. And, t days later it will be $A(t) = 1000\left(1 + \frac{0.05}{356}\right)^t = 1000\left(1 + \frac{1}{7300}\right)^t$.

10 At $x = 3$ the function $f(x)$ has left-hand limit equal to 2 and right-hand limit equal to 1. Therefore $f(x)$ has no limit at $x = 3$.

11 (i) We have

$$\begin{aligned} P(x) &= -[x^2 - 2 \cdot 10 \cdot x] - 75 \\ &= -[x^2 - 2 \cdot 10 \cdot x + 10^2 - 10^2] - 75 \\ &= -[(x-10)^2 - 100] - 75 \\ &= -(x-10)^2 + 100 - 75 \\ &= -(x-10)^2 + 25. \end{aligned}$$

(Note that there are other ways to complete the square.)
(ii) Since $P(x) = -(x-10)^2 + 25$ we see that $P(x)$ takes its maximum value at $x = 10$, which is 25.
(iii) To find the break-even point we solve $P(x) = 0$, or $-(x-10)^2 + 25 = 0$, or $(x-10)^2 = 25$, or $x - 10 = \pm 5$. This gives the break-even points $x = 5$ and $x = 15$.

12 (A) Writing $f(x) = \dfrac{(x-2)(x+2)}{(x-2)(x-5)} \overset{x\neq 2}{=} \dfrac{x+2}{x-5}$, we see that $x = 5$ is a vertical asymptote since $\lim_{x\to 5\pm} \dfrac{x+2}{x-5} = \pm\infty$. Also, we have that $y = 1$ is a horizontal asymptote, since $\lim_{x\to\pm\infty} \dfrac{x+2}{x-5} = 1$.

(B) For $x \neq 3$ we have $\dfrac{x^2+2x-15}{x-3} = \dfrac{\cancel{(x-3)}(x+5)}{\cancel{x-3}} = x + 5$. Thus, $\lim_{x\to 3} f(x) = 8$. Therefore for $f(x)$ to be continuous at $x = 3$ we must have $f(3) = 8$.

13 (A)(i) The revenue $R = q \cdot p = \dfrac{8000q}{q+4}$.
(ii) $\lim_{q\to\infty} \frac{8000q}{q+4} = 8000$.
(B) To find the equilibrium price we solve $D(p) = S(p)$ which gives $-18p + 2,200 = 12p - 800$. Solving for p gives $30p = 3000$ or $p_e = 100$. Then $q_e = 12 \cdot 100 - 800 = 400$.

14 (A) Using the formula $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$, with $A = 100,000, r = 0.05, n = 4$ and $t = 10$ we have the equation $100,000 = P\left(1 + \dfrac{0.05}{4}\right)^{4\cdot 10}$. Solving for P gives $P = 100,000\left(1 + \dfrac{0.05}{4}\right)^{-4\cdot 10}$

(B) Using the formula $P = P_0 b^t$ with $P_0 = 1000$, $P = 9000$ and $t = 2$ we get the equation $9000 = 1000b^2$. Solving it we find $b^2 = 9$ or $b = 3$. (Since $b > 0$, the other root can be ignored.) Thus, we obtain the formula $P(t) = 1000 \cdot 3^t$ for the size of this population at any future time t.