

Department of Mathematics
University of Notre Dame
Math 10250 – Elements of Calculus. 1
Fall 2008

Name:_____

Instructor:_____

Exam II

October 14, 2008

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

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You must record here your answers to the multiple choice problems.

Place an $\times$ through your answer to each problem.

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MC. _____
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Tot. _____

Multiple Choice

1. (5 pts.) Find x for which the slope of the graph of $f(x) = x^2 \ln x$ equals 0.

- (a) $x = 0$ (b) $x = 1$ (c) $x = e$ (d) $x = e^{-1/2}$
(e) $x = -e^{1/2}$

2. (5 pts.) The limit $\lim_{h \rightarrow 0} \frac{99^h - 1}{h}$ is equal to:

- (a) $99 \ln 99$ (b) 99 (c) $\ln 99$
(d) 0 (e) ∞

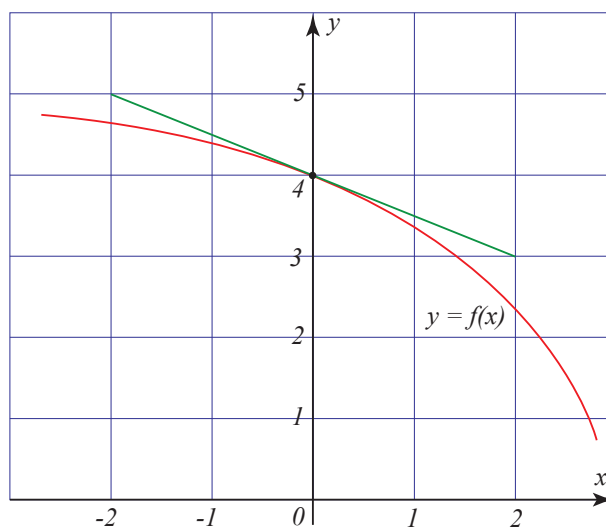
3. (5 pts.) Polonium-210 has a decay constant of 0.004951 with time measured in days. How many days does it take a given quantity of polonium-210 to decay to $1/5$ of its original amount?

- (a) none of these (b) $\frac{\ln 2}{0.004951}$ (c) $\frac{-0.004951}{\ln 5}$
- (d) $\frac{0.004951}{\ln 5}$ (e) $\frac{\ln 5}{0.004951}$

4. (5 pts.) Let $f(x)$ be the function whose graph is displayed in the next figure. Also, let $g(x)$ be the function given by the formula

$$g(x) = e^{x^2+2x}.$$

If $p(x) = f(x)g(x)$, then compute $p'(0)$.



- (a) 7.5 (b) 3.5 (c) 5.5
- (d) -7.5 (e) -3.5

5. (5 pts.) Suppose that $\log_3(a) = 5$. Find the exact value of $\log_3\left(\frac{9}{a^2}\right)$.

- (a) 5 (b) -8 (c) -23
- (d) 2 (e) none of these

6. (5 pts.) A crime scene investigator knows that h hours after death, a body has a temperature of

$$T = T_a + (82 - T_a)(0.6)^h,$$

where T_a is the temperature (in degrees Fahrenheit) of the air surrounding the body. Find the time of death of the body found in a room of 72°F constant air temperature, when its body temperature is 78° .

- (a) 3 (b) none of these (c) 2
(d) 0.5 (e) 1

7. (5 pts.) Let $f(x) = x \ln x$. Find the third derivative $f'''(x)$.

- (a) $x^2 \ln x$ (b) 1 (c) $-\frac{1}{x^2}$ (d) $-\ln x$ (e) $x - \ln x$

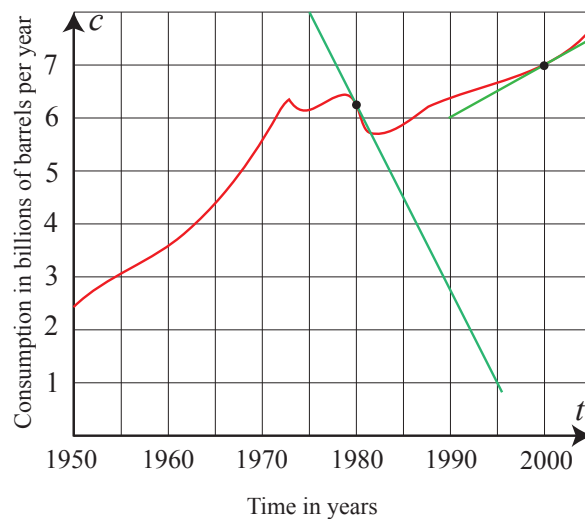
8. (5 pts.) On a nice fall day you are walking along Notre Dame Avenue. Assume that your distance $s(t)$, in meters, from the main circle at time t is given by

$$s(t) = 0.1t^2 + 60t + 30 \quad \text{for } t \geq 0$$

where t is measured in minutes. What is your acceleration (in meter per minute²) after 5 minutes?

- (a) none of these (b) 0.2 (c) 1.2
(d) 0.5 (e) 61

- The next graph displays the rate at which oil was consumed by the U.S. from 1950 to 2005.
9. (5 pts.) Use linear approximation to estimate the oil consumption (in billions of barrels per year) in year 2010.



- (a) 7 (b) 6 (c) none of these
(d) 8 (e) 9

10. (5 pts.) The graph above displays the rate at which oil was consumed by the U.S. from 1950 to 2005. Which of the following statements is **not** true.

- The rate at which consumption changed at $t = 2000$ is $1/10$ billions of barrels per year/per year.
- The rate at which consumption changed at $t = 1980$ is $-7/20$ billions of barrels per year/per year.
- The rate of change of consumption was negative between 1950 and 1970.
- Oil consumption is not differentiable at a time between 1970 and 1980.
- The rate of change of consumption was positive between 1990 and 2000.

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) To buy troubled Mortgage-Backed Securities (MBS) the U.S. government borrows today \$700 billion dollars at the annual interest rate of 4.5% compounded continuously for five years.

(a) (5 pts) What is the US government's investment in MBS after five years?

(b) (5 pts) If at the end of five years the government is able to sell these MBS for \$600 billion dollars, what is the cost to the taxpayers over this period?

12. (10 pts.) In 1908 Ford was selling its Model T car for \$850. Assuming that during the last 100 years inflation was running at the annual rate of 3.15% (compounded continuously) find the price of the Model T car in today's dollars.

13. (10 pts.) The rate of photosynthesis in plants is a function of the intensity of the light. If we let x denote the light intensity (in suitable units) and $p(x)$ the rate of photosynthesis, the function has the form

$$p(x) = \frac{x}{x+1}.$$

Use **only the definition** of the derivative to compute $p'(x)$.

14. (10 pts.) The U.S. debt today is about \$10 trillion dollars. Assuming that the government is not accumulating additional debt and is not making any payments, find how long it will take for the debt to reach the value of \$12 trillion dollars if the annual interest rate paid to the lenders is 5% compounded continuously.

15. (10 pts.) Compute the derivatives of the functions

(a) (5 points) $f(x) = \ln(x^{12} + 2e^{3x} + 3)$

(b) (5 points) $y = \frac{x^3 + 2x}{x^2 - 2}.$

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