Math 10250 Exam 2 Solutions – Fall 2008

1. Applying the product rule we obtain $f'(x) = 2x \ln x + x = x(2\ln x + 1)$. Setting $f'(x) = 0$ gives $x(2\ln x + 1) = 0$. Since x is in the domain of $\ln x$ we must have $x > 0$. Thus, we get the equation $2\ln x + 1 = 0$, or $\ln x = -1/2$, or $x = e^{-1/2}$.

2. Note that $\lim_{h\to 0} \dfrac{99^h - 1}{h}$ is the derivative of function $f(x) = 99^x$ at $x = 0$, hence the answer is $\ln(99)$.

3. Since $1/5 = e^{-0.004951t}$, by taking the natural logarithm, we get answer $t = \dfrac{\ln 5}{0.004951}$.

4. From the picture, $f(0) = 4$ and $f'(0) = -0.5$. By chain rule, $g'(0) = 2$. Therefore, using the product rule, we have $p'(0) = f(0)g'(0) + f'(0)g(0) = 7.5$.

5. $\log_3(\dfrac{9}{a^2}) = \log_3(9) - \log_3(a^2) = 2 - 2\log_3(a) = 2 - 10 = -8$.

6. Substituting for $T = 78$ and $T_a = 72$, we have $78 = 72 + (82 - 72)(0.6)^h$. Solving this gives $h = 1$.

7. $f'(x) = \ln(x) + 1$, so $f''(x) = 1/x$. Hence $f'''(x) = -1/x^2$.

8. Note that $s''(t) = 0.2$.

9. We have $7 + \dfrac{1}{10}(2010 - 2000) = 8$. Therefore the answer is 8.

10. The rate of change between 1950 and 1970 is positive.

11. (a) After five years, it is $700 \cdot e^{0.045\cdot 5} = 876.63$.

(b) $876.63 - 600 = 276.63$.

12. The price in today's dollars is $850 \cdot e^{0.0315\cdot 100} = 19835.65$.

13. Forming the difference quotient (average rate of change) and simplifying gives

$$\frac{p(x+h) - p(x)}{h} = \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} = \frac{(x+h)(x+1) - (x+h+1)x}{h(x+h+1)(x+1)} = \frac{1}{(x+h+1)(x+1)}.$$

Then, letting h go to zero yields the derivative $p'(x) = 1/(x+1)^2$.

14. Since $12 = 10 \cdot e^{0.05\cdot t}$, we have $0.05t = \ln(12/10)$. The answer is $20\ln(1.2) = 3.646$.

15. (a) Using chain rule, we have $f'(x) = \dfrac{1}{x^{12} + 2e^{3x} + 3}(12x^{11} + 6e^{3x})$.

(b) Using quotient rule, we have $\dfrac{dy}{dx} = \dfrac{(3x^2+2)(x^2-2) - (x^3+2x)(2x)}{(x^2-2)^2} = \dfrac{x^4 - 8x^2 - 4}{(x^2-2)^2}$.