Department of Mathematics
University of Notre Dame
Math 10250 – Elements of Calculus I
Fall 2008

Name:______________________________

Instructor:__________________________

Exam III

November 20, 2008

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. **Good luck!**

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature:______________________________________

You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	________
11.	________
12.	________
13.	________
14.	________
15.	________
Tot.	________

Multiple Choice

1. (5 pts.) How many inflection points does $f(x) = 3x^5 - 10x^4$ have?

(a) 0 (b) 1 (c) 2

(d) 3 (e) 4

2. (5 pts.) The critical points of $f(x) = 2x^3 + 3x^2 - 72x + 5$ are:

(a) $x = -2$ and $x = 3$ (b) $x = 2$ and $x = -3$ (c) $x = 3$ and $x = -4$

(d) $x = -5$ and $x = 4$ (e) $x = -2$ and $x = 5$

3. (5 pts.) The slope of the line tangent to $x^2 + y^2 = 25$ at the point $(3, -4)$ is

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{-4}{3}$ (c) 1

(d) $\frac{3}{4}$ (e) $\frac{8}{3}$

4. (5 pts.) The solution of the Initial Value Problem $\frac{dy}{dx} = 1 - \frac{1}{x}$, $y(1) = 3$ is

(a) $y = x - \ln x + 2$ (b) $y = x + \frac{1}{x^2} + 1$ (c) $y = x - \frac{1}{x^2} + 3$

(d) $y = 3 + x - \ln x$ (e) $y = 3 + \ln x$

5. (5 pts.) If a projectile is shot straight up from an elevated platform with an initial velocity of 64 feet per second, then its height, in feet, after t seconds, is given by the formula $h(t) = -16t^2 + 64t + 300$. How high will the projectile go?

(a) 521 feet (b) 112 feet (c) 412 feet

(d) 64 feet (e) 364 feet

6. (5 pts.) A predatory animal expands energy in catching its prey (caloric expense) and gains energy from consuming it (caloric gain). Consider a small predator, a lynx, and let the caloric expense be $E(w) = .1w^4$ and the caloric gain be $G(w) = .4w^3$, where w is the weight of the prey in pounds. What is the optimum prey weight for the lynx?

(a) 1 pound (b) 2 pounds (c) 3 pounds

(d) 4 pounds (e) 5 pounds

7. (5 pts.) The demand function for a certain petroleum product is given by $q = 6000e^{-.05p}$, where p is the price per barrel in dollars and q is the demand in barrels. Suppose that the price is \$10 and is decreasing at the rate of \$2 per day. At what rate is the demand changing?

(a) $\dfrac{600}{\sqrt{e}}$ barrels per day

(b) $-600\sqrt{e}$ barrels per day

(c) $\dfrac{300}{\sqrt{e}}$ barrels per day

(d) $-\dfrac{300}{\sqrt{e}}$ barrels per day

(e) $300\sqrt{e}$ barrels per day

8. (5 pts.) A company manufacturing CD players has determined that the cost of producing x players a week is given by the function $C(x) = 100x + 600$, and the revenue from selling x players a week is given by $R(x) = 500x - x^2$, for $0 \leq x \leq 500$. Assuming that the company sells all the players it produces, how many CD players should the company make each week to maximize its profit?

(a) 100 (b) 200 (c) 300

(d) 400 (e) 500

9. (5 pts.) Suppose that $f(x)$ is a continuous function which is defined for all real numbers x and $f'(x) = (x-1)^2(x-2)^3(x-4)^4$. Which of the following statements is **FALSE**?

(a) The function $y = f(x)$ is increasing at $x = 5$.

(b) The function $y = f(x)$ is decreasing at $x = 3$.

(c) The function $y = f(x)$ has exactly one local minimum.

(d) The function $y = f(x)$ is decreasing at $x = 0$.

(e) The function $y = f(x)$ does not have a local maximum point.

10. (5 pts.) The function $f(x)$ is defined and continuous for all real numbers x except $x = 0$, and $f'(x) = 1 - \dfrac{1}{x}$. Which of the following statements is **FALSE**?

(a) The function $y = f(x)$ is concave up wherever it is defined.

(b) The point $(1, f(1))$ is a local minimum point of $y = f(x)$.

(c) The function $y = f(x)$ is increasing at $x = -1$.

(d) The function $y = f(x)$ does not have any inflection points.

(e) $\lim\limits_{x\to 0^+} f(x) = -\infty$.

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) **Show your work. Circle your answers.**

(a) (5 pts.) Compute $\int \frac{4}{x^5}\,dx$. **Please check your answer.**

(b) (5 pts.) Compute $\int t^2\sqrt{t^3+1}\,dt$. **Please check your answer.**

12. (10 pts.) A box with a square base and a square top is to be built with a volume of 20 cubic feet. The material for the base costs \$3 per square foot, the material for the top costs \$2 per square foot, and the material for the sides costs \$1 per square foot. What is the minimum cost of the box? **Make your work clear. Explain what you are doing. Circle your answer.**

13. (10 pts.) The harvesting period for apples in a region lasts about 7 weeks. The number N of apples harvested during week t is modeled by the function

$$N(t) = 500t^4e^{-t}, \quad \text{for } 0 \leq t \leq 6.$$

Which week yields the largest harvest? **Make your work clear. Explain what you are doing. Circle your answer.**

14. (8 pts.) A chemical spill is contaminating a square region. Each side of the square is growing at at the rate of 2 centimeters per second. How fast is the area of the square growing when each side of the square is 100 centimeters? **Make your work clear. Explain what you are doing. Circle your answer. Be sure to give units.**

15. (12 pts.) Suppose that $f(x)$ is a continuous function defined for all real numbers x with

$$f(0)=0, \quad \lim_{x\to-\infty} f(x)=+\infty, \quad \lim_{x\to\infty} f(x)=0, \quad \text{and} \quad f'(x)=\frac{x-1}{e^x}.$$

(a) Where is $f(x)$ increasing and decreasing?
(b) What are the local minimum points and local maximum points of $y=f(x)$?
(c) Where is $f(x)$ concave up and concave down?
(d) What are the inflection points of $y=f(x)$?
(e) What are the horizontal and vertical asymptotes of $y=f(x)$?
(f) Graph $y=f(x)$.
Your work must be clear. Circle your answers. Make your graph large and clear.