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Math. 10250, Final Exam
Instructor: $\qquad$
December 17, 2008

- Be sure that you have all 14 pages of the test.
- Calculators are allowed for this examination.
- The exam lasts for two hours.
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- Sign the pledge. "As a member of the Notre Dame Community, I will not participate in or tolerate academic dishonesty":

Good Luck!
PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

| 1. (a) | (b) | (c) | (d) | (e) | 17. (a) | (b) | (c) | (d) | (e) |
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| 14. (a) | (b) | (c) | (d) | (e) |  |  |  |  |  |
| 15. (a) | (b) | (c) | (d) | (e) |  |  |  |  |  |
| 16. (a) | (b) | (c) | (d) | (e) |  |  |  |  |  |

## Multiple Choice

1. ( 6 pts.) In 2000 there were about 20,000 units (Peak Kilowatts) of electricity produced from sunlight and in 2005 there were 135,000 units produced ${ }^{1}$. Assuming that the electricity $E$ produced from sunlight is a linear function of time, find a formula for it. (Let $t=0$ be the year 2000.)
(a) $E=20,000 t+23,000$
(b) $E=23,000 t+20,000$
(c) $E=23,000 t-20,000$
(d) $E=-23,000 t+20,000$
(e) $E=20,000 t+135,000$
2.( 6 pts.) The U.S. unemployment rate, $r$, now stands at about 6.7. If for the next few years it is modeled by a quadratic function

$$
r(t)=a(t-h)^{2}+k
$$

and it peaks one year later at $r=7$, then predict its value 3 years from now.
(a) 5.8
(b) 6.0
(c) 7.0
(d) 6.7
(e) 6.3

[^0]3. ( 6 pts.) Find the time needed for an amount of $\$ 500$ to quadruple if it earns an annual interest rate of $5 \%$ compounded continuously.
(a) $30 \ln 4$
(b) $0.5 \ln 4$
(c) $20 \ln 4$
(d) $10 \ln 4$
(e) None of these.
4. ( 6 pts.) A psychologist claims that happiness is a function of the social interactions an individual has. Assume that happiness is modeled by the function
$$
H(x)=\sqrt{x}
$$
where $x$ denotes the number of interactions. Which of the following statement is not true?
(a) The rate of change of happiness tends to zero as the number of interactions becomes too large.
(b) Happiness becomes unboundedly large when the number of interactions becomes too large.
(c) More interactions yield more happiness.
(d) The graph of $H(x)$ is concave up.
(e) The rate of change of happiness decreases as the number of interactions increases.
5. (6 pts.) Find $\lim _{h \rightarrow 0} \frac{\ln (2+h)-\ln 2}{h}$.
(a) 0.5
(b) 2.0
(c) 1.0
(d) 1.5
(e) 0.2
6. ( 6 pts.) The demand for a certain item is $p=-0.02 x+18$, where $x$ denotes the units of quantity sold and $p$ the price in dollars. If the cost function is given by $C(x)=2 x+1200$, then find the maximum profit.
(a) $\$ 3,000$
(b) $\$ 2,000$
(c) $\$ 5,000$
(d) $\$ 4,000$
(e) None of these.
7. ( 6 pts.) Suppose that $f(x)$ is a differentiable function for all real numbers $x$ and such that $f^{\prime}(x)=(x-1)(x-2)^{2}(x-4)$. Which of the following statements is FALSE?
(a) The function $y=f(x)$ is decreasing at $x=3$.
(b) The function $y=f(x)$ is increasing at $x=5$.
(c) The function $y=f(x)$ does not have a local maximum point.
(d) The function $y=f(x)$ has exactly one local minimum.
(e) The function $y=f(x)$ is increasing at $x=0$.
8. ( 6 pts .) Let $Q$ denote the quantity of an item produced when $x$ thousands of dollars are invested for its production. If
$$
Q^{\prime}(x)=300 x^{-2 / 3}
$$
and $Q=1000$ when $x=8$, then find the quantity produced when 27 thousand of dollars are invested.
(a) 1,900
(b) 1,500
(c) 1,700
(d) None of these.
(e) 1,800
9. ( 6 pts .) The graph of a function $g(x)$ and its tangent line at $x=3$ are given below. Find the instantaneous rate of change of $f(x)=\ln [g(x)]$ at $x=3$.
(a) $-3 / 4$
(b) $3 / 2$
(c) $-1 / 2$
(d) $2 / 3$
(e) $1 / 2$

10.(6 pts.) A spherical balloon is being inflated at the rate of 72 cubic inches per minute. Find the rate at which the radius changes at the moment that its volume is $36 \pi$ cubic inches. Recall that the volume of a sphere of radius $r$ is
$$
V=\frac{4 \pi}{3} r^{3}
$$
(a) $\frac{3}{\pi}$
(b) $-\frac{1}{\pi}$
(c) $-\frac{2}{\pi}$
(d) $\frac{2}{\pi}$
(e) $\frac{1}{\pi}$
11. ( 6 pts.) Imagine that you are working for a company designing the car of the future. Your research indicates that this car can evolve over time so that its mileage (in miles per gallon) is modeled by the function
$$
M(t)=250-210 e^{-0.1 t}
$$
which of the following statements is not true.
(a) The graph of $M(t)$ is concave up.
(b) Currently the mileage of this car is 40 .
(c) Mileage increases over time.
(d) The rate of change of $M(t)$ is positive.
(e) As time goes on the mileage approaches 250 .
12.( 6 pts.) In economics utility functions are used to model satisfaction resulting from consumption. An example of a utility function is given by the formula
$$
u(x)=\ln (x+1)
$$
where $x$ denotes the amount of goods (in thousands of dollars per year) consumed by an individual. Which of the following is not true for $u(x)$.
(a) $u(x)$ is an increasing function.
(b) $u^{\prime}(x)$ is decreasing.
(c) $u(x)$ has no critical points.
(d) The limit of $u^{\prime}(x)$ is 0 as $x$ goes to $\infty$.
(e) The graph of $u$ is concave up.
13.( 6 pts.) Using midpoints compute the Riemann sum with four equal subintervals to estimate the value of $\int_{0}^{2} f(x) d x$.
(a) 17
(b) -0.5
(c) None of these.
(d) 0.5
(e) 8.5

14. ( 6 pts.) Compute the area between the curve $y=x^{9}$ and the $x$-axis for $-1 \leq x \leq 1$.
(a) $-1 / 10$
(b) $-1 / 5$
(c) $1 / 5$
(d) 0
(e) $1 / 10$
15. ( 6 pts.) The figure below displays the supply curve $p=S(x)$ for an item, where $p$ stands for the price and $x$ for the quantity. Also, displayed is its tangent line at $x=3$. Use linear approximation to estimate the price at the supply level of $x=3.2$.
(a) 2.2
(b) -2.1
(c) 3.1
(d) 2.1

(e) -2.2
16. ( 6 pts .) The figure below displays the demand curve $p=D(x)$ for an item, where $p$ stands for the price and $x$ for the quantity. Also displayed is its tangent line at $x=2$. Use the product rule to compute the marginal revenue at $x=2$.
(a) 1
(b) 2
(c) 5
(d) 4

(e) 3
17. ( 6 pts .) The figure below displays the graph of a continuous function $f(x)$ defined over the interval $[-1,2]$ Also, it indicates the areas between the graph and the $x$-axis. Compute the following expression $\int_{-1}^{1} f(x) d x+\int_{1}^{2} f(x) d x$.
(a) -20
(b) -10
(c) 10
(d) 17.5
(e) 20

18.( 6 pts.) Compute the indefinite integral $\int \frac{1}{x(x-1)} d x$. (Use partial fractions!)
(a) None of these.
(b) $\quad \ln \left|\frac{1}{x(x-1)}\right|+c$
(c) $\quad \ln \left|\frac{x-1}{x}\right|+c$
(d) $\ln |x(x-1)|+c$
(e) $\quad \ln \left|\frac{x}{x-1}\right|+c$
19. ( 6 pts.) You want to construct a rectangular box with a square base and open top of volume 32 cubic inches. Also, you want to minimize its surface area. What is the minimum area that such a box can have?
(a) 16
(b) 48
(c) 80
(d) 64
(e) 128
20. ( 6 pts.) The speed of a plane at the moment its wheels touch the ground is 3 miles per minute. If the plane decelerates at a constant rate and reaches a complete stop in half a minute, compute the distance (in miles) it traveled during this time.
(a) $1 / 4$
(b) $7 / 4$
(c) $5 / 4$
(d) $9 / 4$
(e) $3 / 4$
21. (6 pts.) The marginal profit for an item is given by $M P(x)=-0.2 x+30$. Compute the total change in profit when the production level $x$ changes from 50 to 100 units.
(a) 950
(b) 850
(c) 650
(d) None of these.
(e) 750
22.(6 pts.) Compute the indefinite integral $\int x e^{-x} d x$. (Use integration by parts!)
(a) $-x e^{-x}+c$
(b) $-x e^{-x}-e^{-x}+c$
(c) $x e^{-x}+c$
(d) $e^{-x}+c$
(e) $x e^{-x}+e^{-x}+c$
23. ( 6 pts.) Choose the number $k$ so that the integral
$$
\int_{0}^{3} x^{k} e^{x^{4}} d x
$$
can be computed by using a substitution.
(a) 3
(b) 5
(c) 2
(d) 4
(e) 1
24. ( 6 pts.) Compute the area between the curves $y=x^{2}$ and $y=\sqrt{x}$ for $0 \leq x \leq 1$.
(a) $1 / 2$
(b) $2 / 3$
(c) $1 / 3$
(d) 2
(e) $-1 / 3$
25. ( 6 pts.) The Figure below displays the gas price in the U.S. for the recent past. Use the mid-point rule with $n=6$ to estimate the average gas price for the two year period starting on $11 / 29 / 2006$ and ending on $11 / 29 / 2008$.
(a) 3.23
(b) 3.43
(c) 3.50
(d) 3.00
(e) 3.11


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[^0]:    ${ }^{1}$ Energy Information Administration

