# Math 1250 Final Exam Solutions – Fall 2008

**1.)** We need to find the equation of the line passing from the points ($t_1 = 0,\ E_1 = 20,000$) and ($t_2 = 5,\ E_2 = 135,000$). The slope of this line is $\frac{\Delta E}{\Delta t} = \frac{135,000-20,000}{5-0} = \frac{115,000}{5} = 23,000$. Therefore busing the point-slope formula we get $E - 20,000 = 23,000(t-0)$ or $E = 23,000t + 20,000$.

**2.)** Since the maximum value of $r$ is 7 and occurs when $t = 1$, we conclude that $h = 1$ and $k = 7$. Thus $r(t) = a(t-1)^2 + 7$. Since $6.7 = r(0) = a(0-1)^2 + 7 = a + 7$ we get $a = 6.7 - 7 = -0.3$. This $r(t) = -0.3(t-1)^2 + 7$. Letting $t = 3$ gives $r(3) = -0.3(3-1)^2 + 7 = -(0.3)4 + 7$, or $\boxed{r(3) = 5.8}$.

**3.)** Using the formula $FV = PVe^{rt}$, which in our situation takes the form $2,000 = 500e^{0,05t}$ and solving for $t$ gives $4 = e^{0.05t}$ or $\ln 4 = 0.05t$ or $\boxed{t = \dfrac{\ln 4}{0.05}}$.

**4.)** The first derivative of the "happiness" function is $H'(x) = \frac{1}{2}x^{-1/2}$ which is positive for all $x > 0$. Thus $H(x)$ is increasing. The second derivative of $H(x)$ is $H"(x) = -\frac{1}{4}x^{-3/4}$, which is negative for $x > 0$. Thus the graph of $H(x)$ is concave down (not concave up as it is stated in one of the answers).

**5.)** This limit is the definition of the derivative of $f(x) = \ln x$ at $x = 2$. Since $f'(x) = 1/x$ we have that this limit is equal to $1/2 = 0.5$.

**6.)** The revenue is given by $R = x.p = x(-0.02x + 18) = -0.02x^2 + 18x$. Therefore the profit function is $P(x) = R(x) - C(x)$ or $P(x) = -0.02x^2 + 18x - (2x + 1200)$ or $P(x) = -0.02x^2 + 16x - 1200$. Taking the derivative gives $P'(x) = -0.04 + 16$. Solving $P'(x) = 0$ or $-0.04x + 16 = 0$ gives $x = 16/0.04$. Thus $x = 400$ is the only critical point. Looking at the sign of $P'(x)$ we see that it goes from $+$ to $-$. Thus the maximum value of the profit is equal to $P(400) = -0.02(400)^2 + 16(400) - 1200 = 2,000$.

**7.)** Finding the sign of derivative we have:
- $f'(x) > 0$ for $x < 1$ or $x > 4$. Thus $f(x)$ is increasing in these intervals.
- $f'(x) < 0$ for $1 < x < 2$ or $2 < x < 4$. Thus $f(x)$ is decreasing in these intervals.
- $f'(1) = f'(2) = f'(3) = 0$. Thus $x = 1, 2, 4$ are critical points.
- At $x = 1$ there is a local maximum and at $x = 4$ there is a local minimum, which means that (c) is false.

**8.)** Taking the integral of both sides of the equation $Q'(x) = 300x^{-2/3}$ we find that $Q(x) = \int 300x^{-2/3}dx = 900x^{1/3} + c$. We can find $c$ by pluging $Q = 1000$ and $x = 8$ into the above equation: $1000 = 1800 + c$ or $c = -800$. Therefore $Q(x) = 900x^{1/3} - 800$, and $Q(27) = 900 \cdot 3 - 800 = 1900$.

**9.)** First we find derivative of $g(x)$. Reading the picture we have $g'(3) = \frac{\Delta y}{\Delta x} = \frac{4-2}{1-3} = -1$. Now applying the chain rule we have $f'(x) = \frac{1}{g(x)} \cdot g'(x)$. Evaluating at $x = 3$ gives $f'(3) = g'(3)/g(3) = \boxed{-1/2}$.

**10.)** Differentiating both sides of the equation and using the chain rule gives $\frac{dV}{dt} = \frac{4\pi}{3}3r^2\frac{dr}{dt}$. Next solving the equation $36\pi = \frac{4\pi}{3}r^3$, we find $r^3 = 27$ or $r = 3$. Therefore when the volume is $36\pi$ then the radius is 3. Since at this moment $\frac{dV}{dt} = 72$ we have the equation $72 = \frac{4\pi}{3}3 \cdot 3^2\frac{dr}{dt}$, which solved for $\frac{dr}{st}$ gives $\boxed{\dfrac{dr}{dt} = \dfrac{2}{\pi}}$.

**11.)** We have $M'(t) = 21e^{-0.1t} > 0$, which means mileage is an increasing function of time. Also, we have $M''(t) = -2.1e^{-0.1t} < 0$, which means the graph of $M(t)$ is concave down (<u>not</u> concave up).

**12.)** Differentiating the utility function we see that $u'(x) = \frac{1}{(x+1)} > 0$, which means that $u(t)$ is increasing. Since $u''(x) = \frac{-1}{(x+1)^2} < 0$ we have that $u(x)$ is concave down (not up).

**13.)** Using the Riemann sum with $n = 4$ and midpoints and reading the given figure gives $\int_0^2 f(x)\,dx \approx [(-5) + (-3) + 3 + 6](0.5) = 0.5$.

**14.)** Since the graph is below the $x$-axis when $-1 \le x \le 0$, we have that the area between the graph and the $x$-axis for $-1 \le x \le 1$ is equal to $-\int_{-1}^{0} x^9\,dx + \int_0^1 x^9\,dx = -\frac{1}{10}(0^{10} - (-1)^{10}) + -\frac{1}{10}(1^{10} - 0^{10}) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$.

**15.)** Reading the picture of the supply curve we find that $S'(3) = \frac{\Delta p}{\Delta x} = \frac{2-1}{3-1} = \frac{1}{2}$. Next, using the point-slope formula with $(x_1 = 3, p_1 = 2)$ and $m = \frac{1}{2}$ gives $p - 2 = \frac{1}{2}(x-3)$, or $p = 2 + \frac{1}{2}(x-3)$. Therefore $S(3.2) \approx 2 + \frac{1}{2}(3.2 - 3) = 2 + 0.1 = \boxed{2.1}$.

**16.)** The revenue is given by $R = x \cdot p$ or $R = x \cdot D(x)$. Applying the product rule gives $R'(x) = 1 \cdot D(x) + xD'(x)$, which evaluated at $x = 2$ gives $R'(2) = D(2) + 2 \cdot D'(2)$. From the given figure we obtain $D(2) = 2$ and $D'(2) = \frac{\Delta p}{\Delta x} = \frac{-1}{2} = -\frac{1}{2}$. Therefore $R'(2) = 2 + 2 \cdot (-\frac{1}{2}) = \boxed{1}$.

**17.)** We have $\int_{-1}^{1} f(x)\,dx + \int_1^2 f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_0^2 f(x)\,dx = 5 - 15 = \boxed{-10}$.

**18.)** First we find $A$ and $B$ such that $\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$ or $1 = A(x-1) + Bx$. Setting $x = 0$ gives $A = -1$. Next, setting $x = 1$ gives $B = 1$. Thus $\displaystyle\int \frac{1}{x(x-1)}\,dx = \int (\frac{1}{x-1} - \frac{1}{x})\,dx = \ln|x-1| - \ln|x| + c = \ln|\frac{x-1}{x}| + c$.

**19.)** If $x$ is the side of the square base and $y$ is the height of the box then its area is $A = x^2 + 4xy$ and its volume is $V = x^2y$. Since $x^2y = 32$ we get $y = \frac{32}{x^2}$. Thus the area can be expressed only as a function of $x$ to obtain $A = x^2 + 4x \cdot \frac{32}{x^2}$ or $A = x^2 + \frac{128}{x}$. Taking the derivative gives $A' = 2x - \frac{128}{x^2}$. Setting it equal to zero we obtain $2x - \frac{128}{x^2} = 0$ or $x^3 = 64$ or $x = 4$. Since $A'' = 2 + 2 \cdot \frac{128}{x^3} > 0$, we conclude that $x = 4$ yields minimum area which is equal to $A(4) = 4^2 + \frac{128}{4} = \boxed{48}$.

**20.)** Constant deceleration means $\frac{dv}{dt} = a$, which gives $v = at + c$. Since $3 = v(0) = 0 + c$ we have $v(t) = at + 3$. Since the plane stops in half a minute we have $v(1/2) = a \cdot \frac{1}{2} + 3 = 0$ or $a = -6$. Thus $v = -6t + 3$. In terms of the position $s(t)$ this equation reads $\frac{ds}{dt} = -6t + 3$. Integrating we obtain $s(t) = -3t^2 + 3t + c$. Since $0 = s(0) = c$ we have $s(t) = -3t^2 + 3t$. Thus the distance traveled is equal to $s(1/2) = -3/4 + 3/2 = \boxed{3/4}$.

**21.)** We have $P(200) - P(100) = \int_{50}^{100} (-0.2x + 30)\,dx = (-0.1x^2 + 30x)\Big|_{100}^{200} = [-0.1(100)^2 + 30 \cdot 100] - [-0.1(50)^2 + 30 \cdot 50] = 750$.

**22.)** Letting $dv = e^{-x}dx$ and $u = x$ we obtain $du = dx$ and $v = -e^{-x}$. Now using integration by parts we have $\int xe^{-x}\,dx = -xe^{-x} - \int(-e^{-x}))\,dx = -xe^{-x} - e^{-x} + c = -(1+x)e^{-x} + c$.

**23.)** Letting $u = x^4$ gives $du = 4x^3dx$. This means that we must choose $k = 3$.

**24.)** Since $\sqrt{x} \ge x^2$ for $0 \le x \le 1$, the area is expressed by the integral

$$\int_0^1 (x^{1/2} - x^2)\,dx = \Big(\frac{x^{1/2+1}}{1/2+1} - \frac{x^{2+1}}{2+1}\Big)\Big|_0^1 = \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}.$$

**25.)** The average gas price is equal to $[2.18 + 3.20 + 2.80 + 3.00 + 3.90 + 3.60] \cdot \frac{1}{6} = \frac{18.78}{6} \approx 3.11$.