Department of Mathematics
University of Notre Dame
Math 10-250 – Bus. Calc. 1
Spring 2008

Name: ______________________________

Instructor: ____________________________

Exam I

February 5, 2008

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. May the force be with you.

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You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

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7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	________
11.	________
12.	________
13.	________
14.	________
Tot.	________

Multiple Choice

1. (5 pts.) The population of a bacteria $P(t)$ (in millions) is given by

$$P(t) = \frac{20}{5 - t}$$

where $0 \leq t < 5$ is time in hours. Which of the statements below is FALSE?

(a) The P(t) is always increasing for $0 \leq t < 5$.

(b) The P(t) has a vertical asymptote.

(c) The population increases in size **unboundedly** as time t approaches 5 hours.

(d) The initial population is 4 million.

(e) The population cannot exceed 20 million.

2. (5 pts.) Find the natural domain of the function $f(x) = \dfrac{\sqrt{x-5}}{x+1}$.

(a) $x \leq 5$ (b) $x \geq 5$ (c) $x \neq -1$

(d) $x > 5$ (e) $x \neq -1, 5$

3. (5 pts.) The supply S (in thousand of units) of a brand of chocolate varies with its price p (in dollars) according to the function:

$$S(p) = 0.5p + 4$$

If the price of the chocolate is **increased** by \$2, what would happen to its supply?

(a) Its supply would <u>decrease</u> by 1 thousand units.

(b) Its supply would <u>decrease</u> by 5 hundred units.

(c) Its supply would <u>increase</u> by 1 thousand units.

(d) Its supply would <u>increase</u> by 2 thousand units.

(e) There is not enough information to determine how its supply would change.

4. (5 pts.) Compute the following limit: $\lim_{x \to 2^+} \dfrac{x^2 - 4}{\sqrt{x - 2}}$

(a) 0 (b) 4 (c) Does not exist.

(d) 1 (e) -4

5. (5 pts.) Let $g(x) = \dfrac{x^2+1}{x^4+x^2+3}$. Which of the following statements is **FALSE**?

(a) The function $g(x)$ is symmetric about the y-axis.

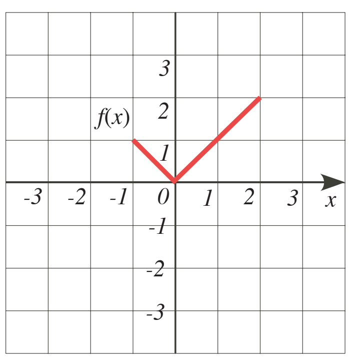
(b) The function $g(x)$ is even.

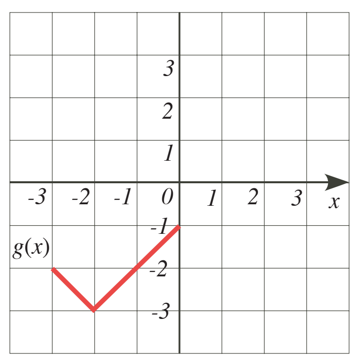
(c) The function $g(x)$ is symmetric about the origin.

(d) The natural domain of $g(x)$ is the set of all real values.

(e) The value of $g(x)$ approaches zero as x gets unboundedly large.

6. (5 pts.) The graphs of $f(x)$ and $g(x)$ are given below.





Which of the following expression describes the relationship between $f(x)$ and $g(x)$?

(a) $g(x) = f(x-2) - 3$ (b) $g(x) = f(x-3) + 2$ (c) $g(x) = f(x+2) + 3$

(d) $g(x) = f(x+2) - 3$ (e) $g(x) = f(x+3) - 2$

7. (5 pts.) Assume that $f(x)$ is a continuous function on the interval $[-2, 3]$ with the following table of values

x	-2	-1	0	1	2	3
$f(x)$	1	-1	-2	-3	-1	2

In which of the following intervals

I. $[-2, -1]$ **II.** $[-1, 0]$ **III.** $[0, 1]$ **IV.** $[1, 2]$ **V.** $[2, 3]$

can you be sure that the function $f(x)$ has a root?

(a) **II** and **IV** only.

(b) **V** only.

(c) **I** and **II** only.

(d) **I** and **V** only.

(e) **II** and **III** only.

8. (5 pts.) For any $a \neq -1, 1$, find the slope of the line joining $(a, 1)$ and $(1, a^2)$.

(a) $a - 1$ (b) $\dfrac{1}{a-1}$ (c) $-(a+1)$

(d) $a + 1$ (e) $\dfrac{1}{-(a+1)}$

9. (5 pts.) Suppose that on the day when his granddaughter is born, a man invested \$6000 in her name in an account paying 4% interest compounded **monthly**. How much will there be on her 21st birthday?

(a) $6000\left(1+\frac{0.04}{12}\right)^{252}$ (b) $6000\left(1+\frac{0.04}{12}\right)^{12}$ (c) $6000\left(1+\frac{0.04}{12}\right)^{21}$

(d) None of the above. (e) $6000\,(1+0.04)^{21}$

10. (5 pts.) Let $f(x)$ be the function whose graph is shown below. Which of the following statements is **FALSE**?

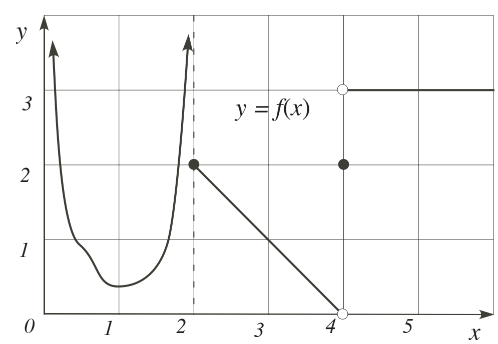
(a) $f(2)=2$.

(b) $\lim_{x\to 2^-} f(x)=+\infty$.

(c) $\lim_{x\to 4} f(x)$ exists.

(d) $f(x)$ is continuous at $x=3$.

(e) $f(x)$ is not continuous at $x=2$.



Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) The profit, in thousands of dollars, from the sales of a certain ipod accessory is given by the formula

$$P(x) = -2x^2 + 12x - 8$$

where x is the number of dozens of accessory sold.

(i) (6 pts.) By completing the square, write $P(x)$ in the form $P(x) = a(x-h)^2 + k$. Show clearly all your steps.

(ii) (4 pts.) Write the maximum profit and the value of x when profit is maximum?

Maximum profit $\stackrel{?}{=}$ ____________ when $x \stackrel{?}{=}$ ________.

(iii) (2 pts.) What is the **fixed** cost of the producing the accessory?

12. (12 pts.)
Part A. Find the **equations** of **all** vertical and horizontal asymptotes of the following functions. If there is none, circle "NONE".

Equations

(i) $f(x) = \dfrac{x^2 - 9}{x^2 - x - 6}$ Vertical: ______________________ NONE

Horizontal: ______________________ NONE

Part B. (Independent from Part A.) Find the value of c such that the function $f(x)$ below is continuous at $x = -1$:

$$f(x) = \begin{cases} \dfrac{x^2 - 5x - 6}{x^2 - 1}, & \text{if } x \neq -1, 1 \\ c, & \text{if } x = -1 \end{cases}$$

Explain your work clearly using the limit definition of continuity.

Answer: __________________

13. (12 pts.) (A) The price function for a model of jeans is

$$p = \frac{12}{x+2}$$

in thousands of dollars and x is the number of jeans sold in units of hundreds. Suppose manufacturing has a fixed cost of three thousands, and the cost of manufacturing one unit of the jeans costs two thousand dollars, write down the following functions

(i) The revenue function $R(x)$.

(ii) The cost function $C(x)$.

(iii) The profit function $P(x)$.

(B) (Independent of A.) How much money should be put in an account paying 4% interest, compounded quarterly (four times a year), in order to have \$10,000 ten years from now?

14. (12 pts.)
(A) **Without using a calculator,** find the equilibrium point for the demand function

$$D(q) = (q-5)^2 + 1 \quad \text{and} \quad S(q) = q + 8 \quad \text{for } 0 < q < 5$$

Find the equilibrium price p_e and equilibrium quantity q_e.

(B) (Independent of A.) A colony of fruit flies is growing exponentially. In 2 hours, the population grew from 1000 to 3000. Write a formula for the size of the population $P(t)$ of the colony of fruit flies as a function of time t in hours. (Hint: Write $P(t) = a \cdot b^t$)

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Math 10250 Exam 1 Solutions – Spring 2008

1. $\lim_{t\to 5^-} P(t) = \lim_{t\to 5^-} \dfrac{20}{5-t} = +\infty$. Therefore the population **will** exceed 20 million.

2. $f(x)$ is defined provided $x - 5 \geq 0$ and $x \neq -1$. So $x \geq 5$ (this already excludes -1).

3. Slope of the graph of S is $0.5 = \dfrac{1}{2}$. So demand S increases as price p increases. Moreover, a \$2 **increase** in p will **increase** S by 1 thousand units.

4. $\lim_{x\to 2^+} \dfrac{x^2-4}{\sqrt{x-2}} = \lim_{x\to 2^+} \dfrac{(x-2)(x+2)}{\sqrt{x-2}} = \lim_{x\to 2^+} \sqrt{x-2}\cdot(x+2) = 0$

5. $g(-x) = \dfrac{(-x)^2+1}{(-x)^4+(-x)^2+1} = \dfrac{x^2+1}{x^4+x^2+1} = g(x)$ so $g(x)$ is an even function. Therefore the graph of $g(x)$ is symmetric about the y-axis. The graph of $g(x)$ is **NOT** symmetric about the origin.

6. The graph of $g(x)$ is obtained from the graph of $f(x)$ by translating the graph of $f(x)$ two units to the left horizontally, and three units down vertically. Therefore $g(x) = f(x+2) - 3$

7. Observe that $f(-2) < 0 < f(-1)$, and $f(2) < 0 < f(3)$. By the Intermediate value theorem, there must be a zero between $[-2,-1]$, and $[2,3]$ only.

8. Slope $= \dfrac{a^2-1}{1-a} = \dfrac{(a-1)(a+1)}{-(a-1)} = -(a+1)$.

9. Principal $P = 6000$, interest rate $r = 4\% = 0.04$, number of compounding $n = 12$, and maturity time $t = 21$ years. Balance $= 6000\left(1+\dfrac{0.04}{12}\right)^{(12)(21)} = 6000\left(1+\dfrac{0.04}{12}\right)^{252}$.

10. $\lim_{x\to 4^-} f(x) = 0$ and $\lim_{x\to 4^+} f(x) = 3$. Therefore $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$. So $\lim_{x\to 4} f(x)$ does **NOT** exist.

11. (i) $P(x) = -2x^2 + 12x - 8 = -2(x^2-6x) - 8 = -2(x^2-6x+3^2-3^2) - 8 = -2((t-3)^2-9) - 8$
$= -2(t-3)^2 + 18 - 8 = -2(t-3)^2 + 10$
(ii) Maximum profit = 10 thousand dollars when $x = 3$ dozens.
(iii) $P(0) = R(0) - C(0) = -C(0) = -8 \Rightarrow C(0) = 8$ thousand dollars.

12. (A) $f(x) = \dfrac{x^2-9}{x^2-x-6} = \dfrac{(x-3)(x+3)}{(x-3)(x+2)} = \dfrac{x+3}{x+2}$ so there is one vertical asymptote $x = -2$. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{x^2-9}{x^2-x-6} = 1$ and $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \dfrac{x^2-9}{x^2-x-6} = 1$. Therefore the is one horizontal asymptote $y = 1$.

(B) For $f(x)$ to be continuous at $x = -1$, $f(-1) = \lim_{x\to-1} \dfrac{x^2-5x-6}{x^2-1} = \lim_{x\to-1} \dfrac{(x-6)(x+1)}{(x-1)(x+1)} = \lim_{x\to-1} \dfrac{x-6}{x-1} = \dfrac{-7}{-2} = \dfrac{7}{2}$. Therefore $c = 7/2 = 3.5$

13. (A) (i) Revenue = price $\cdot$ quantity. $R(x) = p\cdot x = \dfrac{12x}{x+2}$.

(ii) Cost $= C(x) = 2x + 3$ (in thousands of dollars).
(iii) Profit = Revenue − Cost. $P(x) = R(x) - C(x) = \dfrac{12x}{x+2} - (2x+3) = \dfrac{12x}{x+2} - 2x - 3$.

(B) Let P be the principal. Then $\$10000 = P\left(1+\dfrac{0.04}{4}\right)^{4(10)} = P(1.01)^{40} \Rightarrow P = 10000(1.01)^{-40} = \$6716.53$.

14. (A) At equilibrium, $D(q) = S(q) \Rightarrow (q-5)^2+1 = q+8 \Rightarrow q^2-10q+25+1 = q+8 \Rightarrow q^2-10q+26-q-8 = 0 \Rightarrow q^2-11q+18 = 0 \Rightarrow (q-2)(q-9) = 0 \Rightarrow q = 2$ and $q = 9$ (Rejected because $0 < q < 5$). Therefore $q_e = 2$ and $p_e = S(q_e) = 2 + 8 = 10$.
(B) Set $P(t) = a\cdot b^t$. Given $P(0) = 1000$. Then $P(0) = a\cdot b^0 = a = 1000$. Also $P(2) = 2000$. Then $1000\cdot b^2 = 3000 \Rightarrow b^2 = 3 \Rightarrow b = \sqrt{3}$. Therefore $P(t) = 1000(\sqrt{3})^t = 1000(3)^{t/2}$.

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Multiple Choice

1. (5 pts.) Suppose an account pays 6% annual interest compounded **quarterly**. Find the time required for the balance in the account to **triple**.

(a) $\dfrac{\ln 3}{\ln(1.06)}$

(b) $\dfrac{\ln 3}{12\ln(1.005)}$

(c) $\dfrac{\ln 3}{0.06}$

(d) $\dfrac{\ln 3}{4\ln(1.015)}$

(e) Cannot be determined.

2. (5 pts.) Find the limit $\lim\limits_{h\to 0} \dfrac{\dfrac{1}{(3+h)^2} - \dfrac{1}{3^2}}{h}$.

(a) Does not exist.

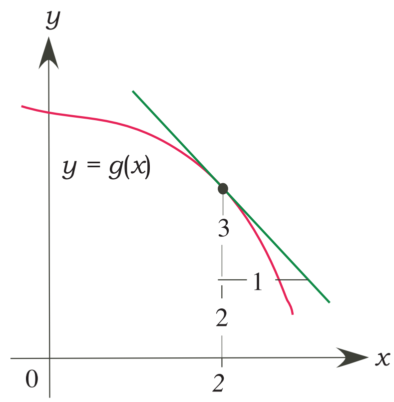
(b) $\dfrac{2}{27}$

(c) $-\dfrac{1}{9}$

(d) $-\dfrac{2}{27}$

(e) $\dfrac{1}{9}$

3. (5 pts.) The graph of the function $g(x)$ is given below. Find the slope of $\dfrac{2x+1}{g(x)}$ at $x = 2$.



(a) $-1/5$ (b) 25 (c) 1

(d) 5 (e) -1

4. (5 pts.) Referring to the same graph of $g(x)$ above, find $f'(2)$ if $f(x) = \dfrac{1-2xg(x)}{x}$.

(a) $\ln 2 + 6$ (b) $-6\frac{1}{4}$ (c) $\ln 2 - 6$

(d) -4 (e) $5\frac{3}{4}$

5. (5 pts.) Let a be a positive number such that

$$\log_a 3 \approx 0.528 \qquad \text{and} \qquad \log_a 5 \approx 0.774.$$

Estimate the value of $\log_a \left(\frac{3a}{25}\right)$.

(a) 0.754

(b) -0.02

(c) 0.02

(d) -1.02

(e) 1.02

6. (5 pts.) Water is flowing into a cylindrical container, whose radius is 2 cm, at a rate of 0.5 cm^3 per second. How fast is the height of the water rising?
(Volume of a cylinder is given by $\pi r^2 h$)

(a) $\frac{1}{2\pi}$ cm/s

(b) 2π cm/s

(c) $\frac{1}{8\pi}$ cm/s

(d) 8π cm/s

(e) $\frac{8}{\pi}$ cm/s

7. (5 pts.) Let $f(x)$ be the function whose graph is shown below. Which of the following statements are **FALSE**?

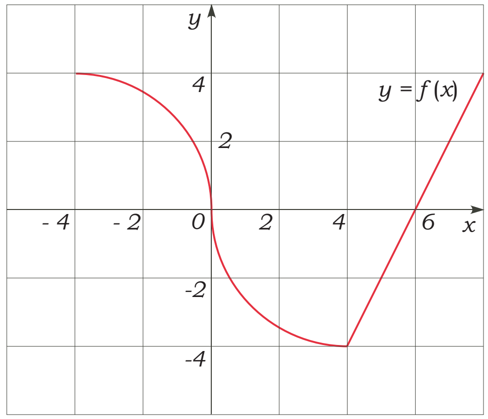
(1) $f'(6) = 2$.

(2) $\lim\limits_{x \to 4} \dfrac{f(x) - f(4)}{x - 4}$ exists.

(3) $f'(0)$ is undefined.

(4) $f(x)$ is continuous at $x = 4$.

(5) The derivative of $f(x)$ is positive at $x = -2$.



(a) (2) and (5) only. (b) (1), (2) and (3) only. (c) (2), (3) and (4) only.

(d) (3) and (4) only. (e) (3) and (5) only.

8. (5 pts.) Find the value(s) of x for which the graph of $y = e^{x^3-9x}$ has a horizontal tangent line.

(a) -3 and 3

(b) $\sqrt{3}$

(c) No such values.

(d) $-\sqrt{3}$ and $\sqrt{3}$

(e) $-3, 0,$ and 3.

9. (5 pts.) Find the linear approximation of $f(x) = x^{2/3} - 3$ near $x = 8$.

(a) $f(x) \approx \frac{1}{3}(x-8)+1$ for x near 8.

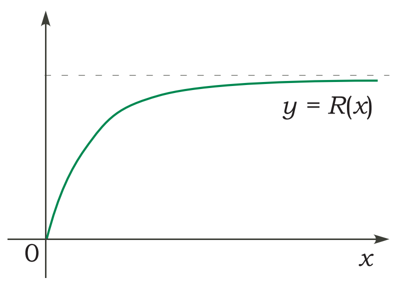
(b) $f(x) \approx \frac{2}{3}x^{-1/3}(x-8)+1$ for x near 8.

(c) $f(x) \approx \frac{1}{3}(x-1)+8$ for x near 8.

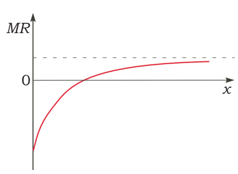
(d) $f(x) \approx \frac{1}{3}(x+8)-1$ for x near 8.

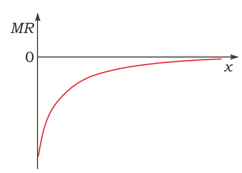
(e) $f(x) \approx \frac{2}{3}x^{-1/3}(x-1)+8$ for x near 8.

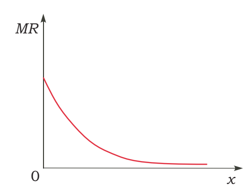
10. (5 pts.) The graph of the revenue $R(x)$ of a company at production level x is given below.

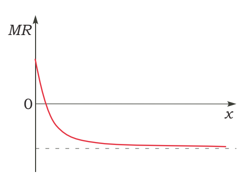


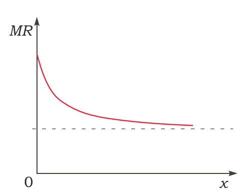
Which of the graphs (a) through (e) **best** describes the **marginal revenue** $MR(x)$?

(a) 

(b) 

(c) 

(d) 

(e) 

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) Consider the function $f(x) = x^2 - x$.

11 (a) Find the value of x for which the tangent to the graph of $f(x)$ is parallel to the line $y = 3x + 2$.

11 (b) Find the average rate of change of $f(x)$ over the interval $1 \leq x \leq (1 + h)$. Simplify your answer as far as possible.

11 (c) Using the limit definition of derivative, find $f'(1)$. You may use the result in 11 (b)

12. (12 pts.) Iodine-131 is a radioactive substance that is used in treating thyroid disorders. 80% of an initial amount will remain after approximately 2.6 days. Find the half-life of Iodine-131.

You should show all steps in your work and you may use the exponential decay model $y = Ae^{kt}$.

13. (12 pts.) A differentiable function $f(x)$ is such that

$$f(1) = 4, \qquad f'(1) = 2, \qquad f(e) = 3 \qquad \text{and} \qquad f'(e) = -1$$

Here e denotes the natural number.

13 (a) Estimate $f(0.8)$ using tangent line approximation near $x = 1$.

13 (b) If $h(x) = e^{f(x)+x^2}$ find $h'(1) \stackrel{?}{=}$ ____________

13 (c) If $k(x) = f(x) \cdot \ln x$ find $k'(e) \stackrel{?}{=}$ ____________

14. (12 pts.)
Part A. An object moving on a straight line has position function

$$s(t) = e^{2t} + 4t^{1/2}.$$

Find the acceleration $a(t)$ of the object at time t.

Part B. The marginal cost of a company is given by

$$MC(x) = -(x+2)^{-2}.$$

Find the marginal revenue $MR(x)$ if the profit function is $P(x) = -x^4 + 9x^2 + \dfrac{1}{2}$.

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3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
Tot.	______

Bus. Calc. 1 Exam 2 Solution – Spring 2008

1. Balance of account at time t, $B(t) = P\left(1 + \dfrac{0.06}{4}\right)^{4t} = P(1.015)^{4t}$. We solve

$$P(1.015)^{4t} = 3P \Rightarrow (1.015)^{4t} = 3 \Rightarrow \ln(1.015)^{4t} = \ln 3 \Rightarrow 4t \cdot \ln(1.015) = \ln 3 \Rightarrow t = \frac{\ln 3}{4\ln(1.015)}.$$

2. The limit is the derivative of $f(x) = 1/x^2 = x^{-2}$ at $x = 3$. $f'(x) = -2x^{-3}$. Therefore

$$\lim_{h\to 0} \frac{\dfrac{1}{(3+h)^2} - \dfrac{1}{3^2}}{h} = f'(3) = -\frac{2}{27}.$$

3. From the graph, $g(2) = 5$ and $g'(2) = -3$. Also $\left(\dfrac{2x+1}{g(x)}\right)' = \dfrac{g(x)(2) - g'(x)(2x+1)}{g(x)^2}$. Therefore the required slope $= \dfrac{g(2)(2) - g'(2)(5)}{g(x)^2} = \dfrac{10+15}{25} = 1$.

4. $f'(x) = \left(\dfrac{1-2xg(x)}{x}\right)' = \left(\dfrac{1}{x} - 2g(x)\right)' = -x^{-2} - 2g'(x)$. Therefore $f'(2) = -2^{-2} - 2(-3) = 5\frac{3}{4}$.

5. $\log_a\left(\dfrac{3a}{25}\right) = \log_a(3a) - \log_a 25 = \log_a 3 + \log_a a - \log_a 5^2 = (0.528) + 1 - 2\log_a 5 = 1.528 - 2(0.774) = -0.02$

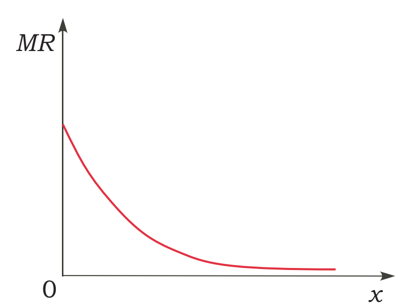
6. $V = \pi r^2 h = 4\pi h$. Both V and h are functions of t. Therefore $V(t) = 4\pi h(t) \Rightarrow \dfrac{dV}{dt} = 4\pi\dfrac{dh}{dt} =$. So $0.5 = 4\pi\dfrac{dh}{dt} \Rightarrow \dfrac{dh}{dt} = \dfrac{1}{8\pi}$.

7. Slope of $f(x)$ at $x = 6$ is $f'(6) = 2$. So (1) is true.
$f'(4) = \lim\limits_{x\to 4} \dfrac{f(x) - f(4)}{x-4}$ does not exist because the graph has a sharp corner at $x = 4$. So (2) is false.
$f'(0)$ is undefined because the tangent line at $x = 0$ is vertical. So (3) is true.
(4) is clearly true.
The slope of the tangent line at $x = -2$ is negative. So (5) is false.

8. $\dfrac{dy}{dx} = (3x^2 - 9)e^{x^3-9x} = 0$. Therefore $x^2 - 3 = 0$ because $e^{x^3-9x} > 0$. So $x = \pm\sqrt{3}$.

9. First find the equation of tangent line at $x = 8$. $f'(x) = \dfrac{2}{3}x^{-1/3}$. Slope at $x = 8$ is $f'(8) = 1/3$. $f(8) = 8^{2/3} - 3 = 1$. Therefore equation of tangent line is $y - 1 = \dfrac{1}{3}(x-8) \Rightarrow y = \dfrac{1}{3}(x-8) + 1$. Linear approximation of $f(x)$ near $x = 8$ is $f(x) \approx \dfrac{1}{3}(x-8) + 1$.

10. The slope of $R(x)$ is always positive. So $MR(x) = R'(x) > 0$. Also the slope of $R(x)$ approaches zero as x increases. Therefore $\lim_{x\to\infty} MR(x) = 0$ so the answer is:



11. (a) Slope of line is 3. Solve $f'(x) = 3$. So $2x - 1 = 3 \Rightarrow x = 2$.

(b) $\dfrac{f(1+h) - f(1)}{h} = \dfrac{(1+h)^2 - (1+h) - 0}{h} = \dfrac{1 + 2h + h^2 - 1 - h}{h} = \dfrac{h^2 + h}{h} = h + 1$.

(c) $f'(1) = \lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h \to 0} (h+1) = 1$

12. $y = Ae^{kt}$. Given that $y(2.6) = 0.8A$. Therefore $Ae^{2.6k} = 0.8A \Rightarrow e^{2.6k} = 0.8 \Rightarrow 2.6k = \ln(0.8) \Rightarrow k = \dfrac{\ln(0.8)}{2.6} =$. So $y = Ae^{\frac{t\ln(0.8)}{2.6}}$.

At half-life $y(t) = 0.5A$. Therefore $Ae^{\frac{t\ln(0.8)}{2.6}} = 0.5A \Rightarrow e^{\frac{t\ln(0.8)}{2.6}} = 0.5 \Rightarrow \dfrac{t\ln(0.8)}{2.6} = \ln(0.5) \Rightarrow t = \dfrac{2.6\ln(0.5)}{\ln(0.8)} =$ 8.076 days.

13. (a) Equation of tangent line at $x = 1$ is $y - 4 = 2(x - 1)$. Therefore $f(x) \approx 2(x-1) + 4$ so $f(0.8) = 2(-0.2) + 4 = 3.6$.

(b) $h'(x) = (f'(x) + 2x)e^{f(x)+x^2}$. Therefore $h'(1) = (f'(1) + 2)e^{f(1)+1} = 4e^5$.

(c) $k'(x) = f(x) \cdot \dfrac{1}{x} + f'(x)\ln x$. Therefore $k'(e) = f(e) \cdot \dfrac{1}{e} + f'(e)\ln e = \dfrac{3}{e} - 1$.

14. (A) Velocity $v(t) = s'(t) = 2e^{2t} + 2t^{-1/2}$. Acceleration $a(t) = s''(t) = 4e^{2t} - t^{-3/2}$.

(B) $P(x) = R(x) - C(x)$ so $MP(x) = MR(x) - MC(x)$. Therefore $-4x^3 + 18x = MR(x) + (x+2)^{-2}$ so $MR(x) = -4x^3 + 18x - (x+2)^{-2}$.

Department of Mathematics
University of Notre Dame
Math 10-250 – Bus. Calc. 1
Spring 2008

Name:________________________________

Instructor:____________________________

Exam III

April 17, 2008

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. May the force be with you.

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature:________________________________

You must record here your answers to the multiple choice problems.
Place an $\times$ through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
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4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	________
11.	________
12.	________
13.	________
14.	________
Tot.	________

Multiple Choice

1. (5 pts.) Find the critical points for the function $f(x) = 2x + \dfrac{8}{x}$.

(a) 2 only.

(b) -2 and 0 only.

(c) 0 only.

(d) -2 and 2 only.

(e) -2, 0 and 2 only.

2. (5 pts.) A graphic artist designing a poster for commercial use is instructed to have **one** inch margins top and bottom, and **two** inches along each side around the printed portion of the poster. It is further specified that the total area of the printed portion is 100 square inches. Suppose the width of the poster is x inches and the length of the poster is y inches. To cut paper cost, the total area of the poster (margin and printed portion) is to be minimized. Which of the following functions of x would you minimize? **Do not solve the rest of the problem!**

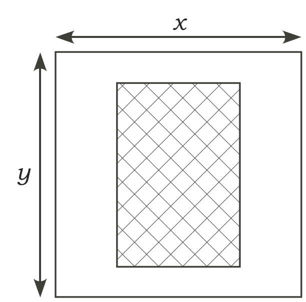
(a) $\dfrac{100}{x-4} + 2$

(b) $\dfrac{100}{x-2} + 4$

(c) $\dfrac{100x}{x-4} + 2x$

(d) None of these

(e) $\dfrac{100x}{x-2} + 4x$



3. (5 pts.) The second derivative of a smooth function $f(x)$ is given by

$$f''(x) = (-x^3 - 2x^2 + 2x)e^x.$$

If $x = 0$ and 1 are critical points of $f(x)$, what can you conclude about $f(x)$ using the **second derivative test**?

(a) Local maximum at $x = 1$ and no conclusion for $x = 0$.

(b) Local maximum at $x = 1$ and local minimum at $x = 0$.

(c) Local minimum at $x = 1$ and no conclusion for $x = 0$.

(d) Local maximum at $x = 1$ and, neither local maximum nor minimum at $x = 0$.

(e) Local minimum at $x = 1$ and, neither local maximum nor minimum at $x = 0$.

4. (5 pts.) Find the value of A for which $F(x) = A(2x - 5)^{10}$ is an antiderivative of

$$f(x) = 3(2x - 5)^9$$

(a) $A = \dfrac{3}{20}$

(b) $A = \dfrac{3}{10}$

(c) $A = 60$

(d) Any value of A.

(e) $A = 30$

5. (5 pts.) The function $f(x)$ whose derivative is given by

$$f'(x) = (x^2 + 2x - 3)e^{\frac{x^3}{3}+x^2-3x}.$$

Given that $x = -3$ and $x = 1$ are critical points of $f(x)$. Use the **first derivative test** to determine which of the following statement is **TRUE**.

(a) $f(x)$ has global maximum at $x = -3$ but no conclusions could be draw about $f(x)$ at $x = 1$.

(b) $f(x)$ has a local minimum at $x = 1$ but a local maximum at $x = -3$.

(c) $f(x)$ has local minimums at both $x = 1$ and $x = -3$.

(d) $f(x)$ has a local maximum at $x = 1$ but a local minimum at $x = -3$.

(e) $f(x)$ has local maximums at both $x = 1$ and $x = -3$.

6. (5 pts.) Find the global maximum and global minimum, if they exist, for the function $f(x) = xe^{-10x}$ for $0.01 \le x < \infty$.

(a) Global minimum is 0; Global maximum $0.1e^{-1}$.

(b) Global minimum is $0.0099e^{-0.099}$; Global maximum $0.11e^{-1.1}$.

(c) No global minimum ; Global maximum is $0.1e^{-1}$.

(d) No global minimum ; Global maximum is $0.099e^{-0.99}$.

(e) Global minimum is $0.01e^{-0.1}$; Global maximum $0.1e^{-1}$.

7. (5 pts.) The figure below (Figure 1) is the graph of the **derivative** of $f(x)$. Which of the following statements are **TRUE**?

(1) $f(x)$ is increasing on $(-\infty, 0) \cup (4, \infty)$.

(2) $f(x)$ is decreasing on $-2 < x < 2$.

(3) $f(x)$ has a local maximum at $x = 0$.

(4) $f(x)$ has critical points at $x = -2$ and $x = 2$.

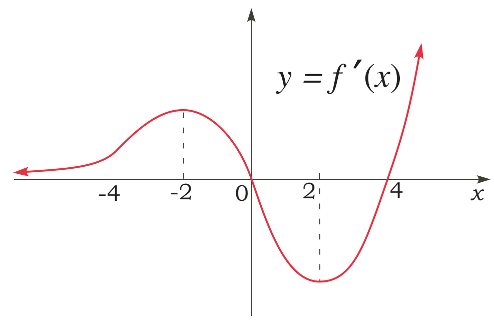


Figure 1

(a) (2) and (3) only. (b) (2), (3) and (4) only. (c) (1), (2) and (3) only.

(d) (1) and (3) only. (e) (1) only.

8. (5 pts.) Still referring to **Figure 1** above, which of the following statement is **TRUE** about $f(x)$? There is only one correct answer.

(a) $f(x)$ has an inflection point at $x = 0$.

(b) None of these.

(c) $f(x)$ is concave up on $(-\infty, -4) \cup (0, \infty)$.

(d) The concavity of $f(x)$ does not change for $0 < x < 4$.

(e) $f(x)$ is concave down on $-2 < x < 2$.

9. (5 pts.) With the substitution $u = 1 + x^3$, the indefinite integral $\int x^2\sqrt{1+x^3}\,dx$ evaluates to

(a) $\dfrac{2u^{3/2}}{3} + C$

(b) $\dfrac{u^{-1/2}}{6} + C$

(c) $\dfrac{u^{-1/2}}{3} + C$

(d) $\dfrac{2u^{3/2}}{9} + C$

(e) None of these.

10. (5 pts.) Evaluate the following indefinite integration

$$\int \frac{x^2 + x - 2}{x}\,dx$$

(a) $\dfrac{\dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x + C}{\dfrac{x^2}{2} + C}$

(b) $1 + \dfrac{2}{x^2} + C$

(c) $\dfrac{x^2}{2} + x - 2\ln|x| + C$

(d) $2x + 1 + C$

(e) None of these.

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) The equation of a curve C is given by $\ln x + xy - 3y^2 = -2$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) Find the equation of the tangent line through the point $(1, 1)$ to the curve C.

12. (12 pts.) Suppose the demand function for a certain product is given by the equation

$$q = 60 - 2\sqrt{x}$$

where $0 \leq x \leq 800$ is the unit price of the product and q is the quantity, in hundreds, sold.

a. Write down the **revenue** function $R(x)$.

b. Use Calculus to find the value of x that maximizes the revenue function $R(x)$. You need to **clearly** show that the value you obtained indeed maximizes the revenue. Here $0 \leq x \leq 800$.

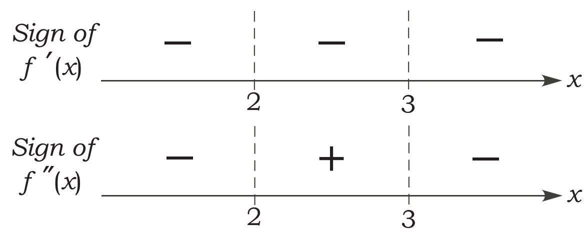
(Picking out an answer with a graphing calculator will not be considered as work for this problem.)

13. (12 pts.) The function $f(x)$ has the following properties:

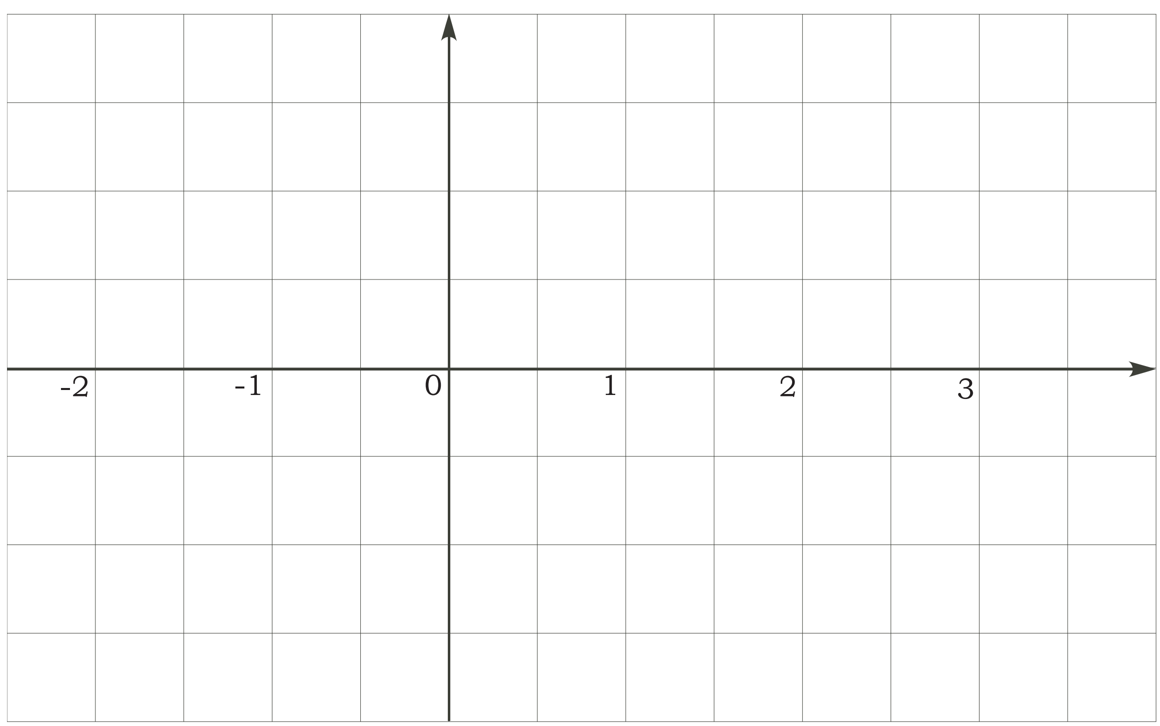
a. $f(x)$ has vertical asymptote at $x = 2$.

b. $f(3) = 0$

c. $f(x)$ has a critical point at $x = 3$ such that $f'(3) = 0$.



Draw a possible graph of $f(x)$ illustrating **ALL** the properties above. Mark clearly the **coordinates of the inflection point(s)** in your graph.



14. (12 pts.) A farmer uses a large tank connected to his irrigation system to distribution 100 gallons of liquid fertilizer to his fields. When irrigation begins, he opens the tank's valve, allowing the fertilizer to leave the tank at a rate of $22e^{-0.2t}$ gallons per minute.

a. Write down an initial value problem that models the amount of fertilizer $V(t)$ in the time t minutes after the farmer begins the irrigation.

(Hint: Your answer must have a differential equation and an initial value)

b. Without using a calculator, solve for the amount of fertilizer in the tank at time t.

Department of Mathematics
University of Notre Dame
Math 10-250 – Bus. Calc. 1
Spring 2008

Name:______________________________

Instructor:__________________________

Exam III

April 17, 2008

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4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	________
11.	________
12.	________
13.	________
14.	________
Tot.	________

Math 10250 Exam III Solutions – Spring 2008

1. $f'(x) = 2 - \dfrac{8}{x^2} = 0 \Rightarrow 2 = \dfrac{8}{x^2} \Rightarrow 2x^2 = 8 \Rightarrow x^2 = 4 \Rightarrow x = -2, 2$ are critical points. Note that $x = 0$ is not a critical point because it is not in the domain of $f(x)$.

2. Printed region has area $= 100 = (x-4)(y-2) \Rightarrow y - 2 = \dfrac{100}{x-4} \Rightarrow y = \dfrac{100}{x-4} + 2$. Therefore the total area of the poster $A = xy = \dfrac{100x}{x-4} + 2x$. Minimize $A(x)$ on $4 < x < \infty$.

3. $f''(0) = 0$ so no conclusions could be drawn. $f''(1) = (-1-2+2)e = -e < 0$ so $f(x)$ has a local maximum at $x = 1$.

4. By definition of antiderivative $F'(x) = f(x)$. Therefore $A \cdot 10(2x-5)^9 \cdot 2 = 3(2x-5)^9$. So $20A = 3 \Rightarrow A = 3/20$.

5. Check sign of $f(x)$ in the regions between critical points. Therefore $f(x)$ has a local minimum at $x = 1$ and a local maximum at $x = -3$.

6. Check endpoints: $f(0.01) = 0.01e^{-0.1} \approx 0.00905$. Check critical points: $f(x) = -10xe^{-10x} + e^{-10x} = (-10x+1)e^{-10x} = 0 \Rightarrow x = 1/10 = 0.1 \Rightarrow f(0.1) = 0.1e^{-1} \approx 0.0368$. Note that $\lim_{x\to\infty} f(x) = 0$. A rough sketch of the graph $f(x)$ shows that global maximum is $f(0.1) = 0.1e^{-1}$, and no global minimum.

7. (1) is true because $f'(x)$ is positive on $(-\infty, 0) \cup (4, \infty)$.

   (2) is not true because $f'(x)$ is positive on $-2 < x < 0$ (f(x) is increasing).

   (3) is true. $f'(0) = 0 \Rightarrow x = 0$ is a critical point. Moreover, using first derivative test, $x = 0$ is a local maximum ($f'(x) > 0$ for $x < 0$ and $f'(x) < 0$ for $x > 0$).

   (4) is not true. $f'(x) \neq 0$ at $x = -2, 2$.

8. The statement "$f(x)$ is concave down on $-2 < x < 2$" is true because $f'(x)$ is decreasing on $-2 < x < 2$ i.e. $f''(x) < 0$ on $-2 < x < 2$.

   The statement "$f(x)$ has an inflection point at $x = 0$" is not true because $f(x)$ is concave down on $-2 < x < 2$ so there is no change in concavity at $x = 0$.

   The statement "$f(x)$ is concave up on $(-\infty, -4) \cup (0, \infty)$" is not true because $f(x)$

   The statement "The concavity of $f(x)$ does not change for $0 < x < 4$" is not true because $f''(x)$ changes signs in $0 < x < 4$.

9. $u = (1 + x^3) \Rightarrow du = 3x^2 dx$. So $\displaystyle\int x^2\sqrt{1+x^3}dx = \int \sqrt{1+x^3} \cdot x^2 dx = \int \frac{u^{1/2}}{3} du = \frac{\frac{u^{3/2}}{3/2}}{3} + C = \frac{2u^{3/2}}{9} + C$.

10. $\displaystyle\int \frac{x^2+x-2}{x}\, dx = \int \left(x + 1 - \frac{2}{x}\right) dx = \frac{x^2}{2} + x - 2\ln|x| + C.$

11. (a) $\dfrac{d}{dx}\left(\ln x + xy - 3y^2\right) = \dfrac{d}{dx}(-2) \Rightarrow \dfrac{1}{x} + x \cdot \dfrac{dy}{dx} + 1 \cdot y - 6y\dfrac{dy}{dx} = 0 \Rightarrow -\dfrac{1}{x} - y = -6y\dfrac{dy}{dx} + x\dfrac{dy}{dx}$

    $\Rightarrow (-6y + x)\dfrac{dy}{dx} = -\dfrac{1}{x} - y \Rightarrow \dfrac{dy}{dx} = \dfrac{-\frac{1}{x} - y}{-6y + x}$

    (b) Slope at $(1,1) = \left.\dfrac{dy}{dx}\right|_{(1,1)} = \dfrac{-1-1}{-6+1} = \dfrac{2}{5}$. So the equation of required tangent line is $y - 1 = \dfrac{2}{5}(x-1)$ i.e. $y = \dfrac{2}{5}x + \dfrac{3}{5}$.

12. (a) Revenue = Price × Quantity. $R(x) = x \cdot q = x(60 - 2\sqrt{x}) = 60x - 2x^{3/2}$.

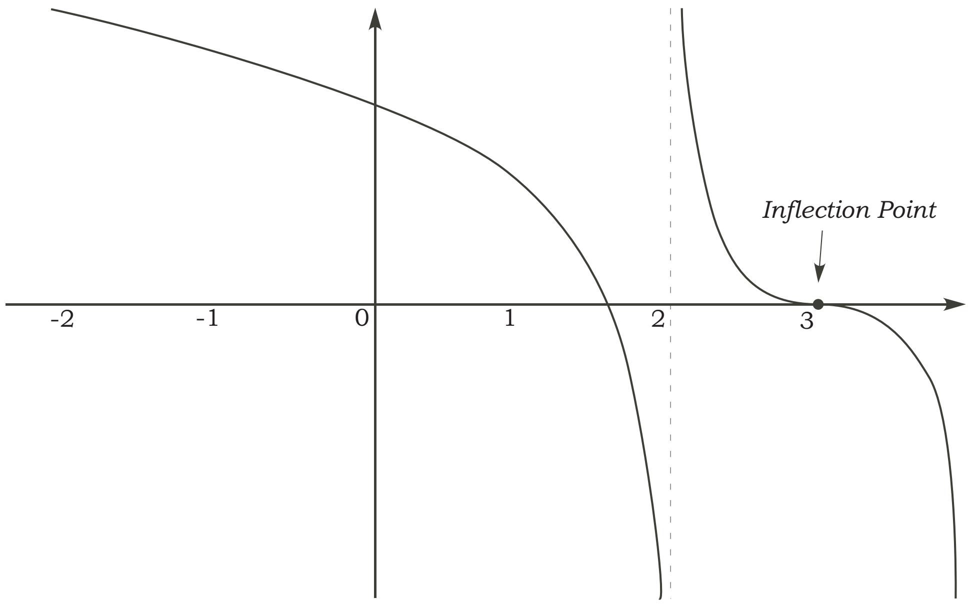
(b) End-points: $R(0) = 0$ and $R(800) = 60(800) - 2(800)^{3/2} = 2745.166$

Critical point: $R'(x) = 60 - 3x^{1/2} = 0 \Rightarrow x^{1/2} = 20 \Rightarrow x = 20^2 = 400$.

$R(400) = 60(400) - 2(400)^{3/2} = 8000$

We are maximizing on a closed and bounded interval $[0, 800]$ so maximum must occur at end-point or critical point. Therefore $x = 400$ gives global maximum.

13.



14. (a) $\dfrac{dV}{dt} = -22e^{-0.2t}; V(0) = 100.$

(b) So $V(t) = \int -22e^{-0.2t}dt = \dfrac{-22}{-0.2}e^{-0.2t} + C = 110e^{-0.2t} + C$. Therefore $V(0) = 110 + C = 100 \Rightarrow C = -10$. Hence, $V(t) = 110e^{-0.2t} - 10$.