Name: _____

Math 10250, Final Exam - Version A Instructor: _____
May 8, 2007

- Be sure that you have all 16 pages of the test.
- Calculators are allowed for this examination.
- The exam lasts for two hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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2.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
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10.	(a)	(b)	(c)	(d)	(e)	26.	(a)	(b)	(c)	(d)	(e)
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14.	(a)	(b)	(c)	(d)	(e)	30.	(a)	(b)	(c)	(d)	(e)
15.	(a)	(b)	(c)	(d)	(e)						
16.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1.(5 pts.) If the balance of a savings account, earning interest at an annual rate of 5% compounded continuously, triples in T years, what is the value of T?

(a) $T = 20 \ln 3$ (b) $T = \dfrac{\ln 3}{\ln 1.05}$ (c) $T = \dfrac{1}{15} \ln 3$

(d) $T = \ln 3 - 0.05$ (e) Cannot be determined.

2.(5 pts.) The demand and supply curves of a certain commodity are given below:

$$S(q) = q^2 + 2, \qquad D(q) = -3q + 6$$

Find the equilibrium price and the equilibrium quantity.

(a) Equilibrium price = 4; Equilibrium quantity = 18

(b) Equilibrium price = 18; Equilibrium quantity = 4

(c) Equilibrium price = 3; Equilibrium quantity = 1

(d) Equilibrium price = 11; Equilibrium quantity = 3

(e) None of these.

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3.(5 pts.) You just won a lottery that awards you 500 thousand dollars. All your money goes into an account paying 8% interest per year, **compounded quarterly**. Calculate the **balance** of your account (in thousands of dollars) after **five** years assuming no withdrawals were made.

(a) $500(1.02)^{20}$

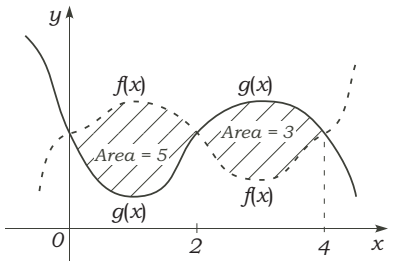
(b) $500e^{0.4}$

(c) $500(1.02)^{5}$

(d) $500(1.08)^{20}$

(e) $500(1.04)^{10}$

4.(5 pts.) The graphs of the functions $f(x)$ (dotted line) and $g(x)$ (solid line) are shown below. What is the **average** of $h(x) = f(x) - g(x)$ for $0 \leq x \leq 4$?



(a) 4 (b) -0.5 (c) 0.5

(d) -2 (e) 2

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5.(5 pts.) The velocity of a car at time t is $v(t) = \dfrac{2}{t+1} + 2e^{2t}$ meters per second. If the initial displacement is 3 meters, find the displacement $s(t)$ for all time t.

(a) $s(t) = \dfrac{2t}{(t^2/2)+t} + e^{2t}$

(b) $s(t) = -2(t+1)^{-2} + 4e^{2t} + 1$

(c) $s(t) = 2\ln|t+1| + e^{2t}$

(d) $s(t) = 2\ln|t+1| + e^{2t} + 2$

(e) $s(t) = 2\ln|t+1| + 2e^{2t} + 1$

6.(5 pts.) Find the natural domain of the function $f(x) = \dfrac{x^2-4}{(x^2+1)\sqrt{3-x}}$.

(a) $x < 3$ (b) $x \neq 3$ (c) $x > 3$

(d) $x \neq -1, 1, 3$ (e) $x \neq 2, -2$

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7.(5 pts.) Applying the integration by parts formula to the definite integral $\int_1^e t^3 \ln t \, dt$ with $u = \ln t$ gives us the expression:

(a) $\dfrac{e^5+4}{5} - \displaystyle\int_1^e t^4 \ln t \, dt$

(b) $\dfrac{e^4}{4} - \displaystyle\int_1^e \frac{t^3}{4} dt$

(c) $\dfrac{e^3}{3} - \displaystyle\int_0^1 \frac{1}{3} e^{3t} dt$

(d) None of these choices.

(e) $\displaystyle\int_0^1 u e^{3u} du$

8.(5 pts.) Using the substitution $u = x^3+3x^2-5$, the integral $\displaystyle\int (x^2+2x)(x^3+3x^2-5)^{10} dx$ evaluates to:

(a) $30u^9 + C$

(b) $\dfrac{u^9}{27} + C$

(c) $\dfrac{u^{11}}{33} + C$

(d) $\dfrac{u^{11}}{11} + C$

(e) None of these choices.

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9.(5 pts.) Find all possible values of k for which the function $f(x) = \begin{cases} \dfrac{x^2-3x+2}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$ is a continuous for all x.

(a) 0

(b) All real k except 1.

(c) 1

(d) -1

(e) No such k exists.

10.(5 pts.) A colony of insects had a population of 5 thousands when first observed. Two days later, the population grew to 20 thousands. Let $P(t)$ be the population (in thousands) of the insects t days after it was first observed. If $P(t)$ is an **exponential** function of t, what is the population when $t = 3$? (Hint: Write $P(t) = a \cdot b^t$)

(a) $P(3) = 40$

(b) $P(3) = 1000$

(c) $P(3) = 8000$

(d) $P(3) = 35$

(e) $P(3) = 320$

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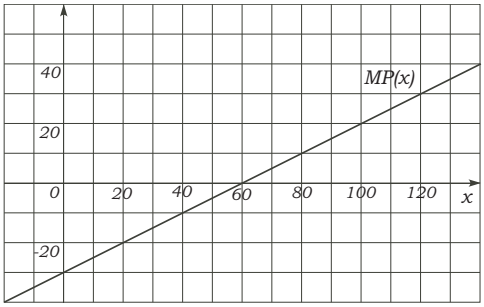
11.(5 pts.) Tom produces 20 surfboards a month. He estimates the marginal profit (in dollars per board) to be given by the straight line graph below.



What will the change in profit be if Tom increases production to 120 surfboards a month?

(a) \$900 (b) \$50 (c) None of these choices.

(d) \$1300 (e) \$500

12.(5 pts.) Find all critical point of the function $f(x) = xe^{2x}$.

(a) -1 (b) $1/2$ (c) 0

(d) $-1/2$ (e) 1

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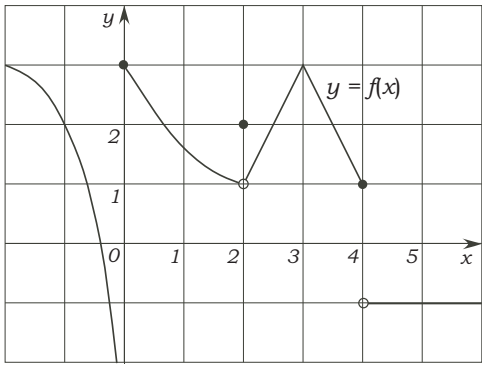
13.(5 pts.) The graph of $y = f(x)$ is given below. Which of the following statements (1) to (5) are **TRUE**?



(1) $\lim_{x\to 0^-} f(x)$ is a real number.

(2) $\lim_{x\to 0^+} f(x)$ does not exist.

(3) $\lim_{x\to 2} f(x) = 1$.

(4) $f(4) = -1$.

(5) $\lim_{x\to 3} f(x)$ exists.

(a) (2), (3) and (4) only.

(b) (3) and (5) only.

(c) (1), (3) and (5) only.

(d) (2) and (4) only.

(e) (1) and (3) only.

14.(5 pts.) Referring to the **same graph as above**, which of the following statement is **TRUE**?

(a) $f(x)$ is continuous at $x = 3$.

(b) $\lim_{h\to 0} \dfrac{f(3+h) - f(3)}{h}$ exists.

(c) $f(x)$ is discontinuous at $x = 1$.

(d) $f(x)$ is differentiable at $x = 2$.

(e) $f(x)$ is continuous at $x = 2$.

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15.(5 pts.) The figure below (Figure 1) is the graph of the **derivative** of $f(x)$. Which of the following statements are **TRUE**?

(1) $f(x)$ is increasing on $-\infty < x < -2$.

(2) $f(x)$ is decreasing on $0 < x < 2$.

(3) $f(x)$ has a local maximum at $x = 0$.

(4) $f(x)$ has critical points at $x = -2$, $x = 0$ and $x = 2$.

(5) $f(x)$ is decreasing on $-1 < x < 1$.

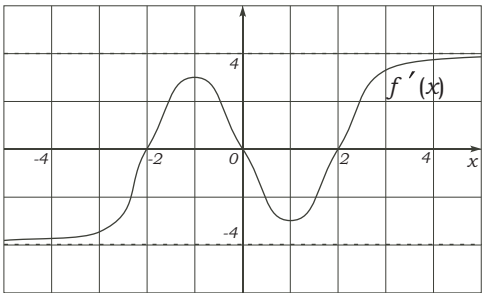

Figure 1

(a) (4) and (5) only. (b) (3) and (5) only. (c) (2), (3) and (4) only.

(d) (1), (2) and (3) only. (e) (2) and (3) only.

16.(5 pts.) Still referring to **Figure 1** above, which of the following statements are **TRUE** about $f(x)$?

(1) $f(x)$ is concave down on $-1 < x < 0$.

(2) $f(x)$ is concave up on $0 < x < 2$.

(3) $f(x)$ has an inflection point at $x = -1$.

(4) $f(x)$ has an inflection point at $x = 0$.

(a) (3) and (4) only. (b) (1), (2) and (4) only. (c) (2), (3) and (4) only.

(d) (1) and (3) only. (e) (1) and (4) only.

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17.(5 pts.) Find $\dfrac{dy}{dx}$ if $y^2 + xy - x^2 = 5$.

(a) $\dfrac{dy}{dx} = \dfrac{2x - y}{2y + x}$

(b) $\dfrac{dy}{dx} = \dfrac{2x - y + 5}{2y + x}$

(c) $\dfrac{dy}{dx} = \dfrac{1}{3}$

(d) $\dfrac{dy}{dx} = \dfrac{-2x + y}{2y + x}$

(e) $\dfrac{dy}{dx} = \dfrac{-y}{2y - x - 5}$

18.(5 pts.) The demand function $D(p)$ (in thousands per month) of a brand of mountain bikes in terms of its price (per bike) is given by

$$D(p) = \frac{50}{2p + 1}.$$

Which of the statements below is **FALSE**?

(a) The demand function has a horizontal asymptote.

(b) The demand function is always decreasing for $p > 0$.

(c) The **revenue** from the sale of bikes increases **unboundedly** as price increases.

(d) The demand increases as price decreases.

(e) The **revenue** from the sale of bikes cannot exceed 25 thousand dollars.

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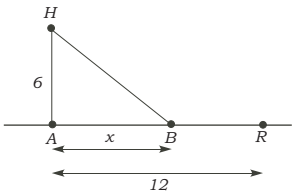
19.(5 pts.) A house H is located in the woods, 6 miles from the nearest point, A, on a straight road. A restaurant, R, is located 12 miles down the road from A. Jack can ride his bike 2 miles per hour in the woods and 10 miles per hour along the road. He decides to ride the bike through the woods to some intermediate point B, x miles from A, and then ride along the road to R. Since he is starving, he wants to minimize his time. Which of the following is the function to be minimized? **Do not solve the rest of the problem!**



(a) $12 + 10x$

(b) $\dfrac{\sqrt{36+x^2}}{2} + \dfrac{12-x}{10}$

(c) $\dfrac{\sqrt{36+x^2}}{2} + \dfrac{x}{10}$

(d) $3 + \dfrac{x}{10}$

(e) $2\sqrt{36+x^2} + 10(12-x)$

20.(5 pts.) Find all values of x where the graph of $f(x) = x^3 - x^2$ has slope $= 1$.

(a) None of these choices.

(b) $x = 0, \quad 2/3$

(c) $x = -1, \quad 1/3$

(d) $x = -2/3, \quad 0$

(e) $x = -1/3, \quad 1$

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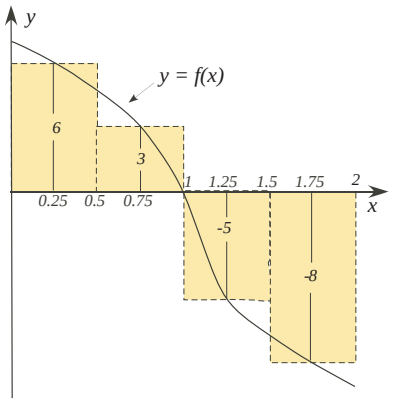
21.(5 pts.) The graph of $f(x)$ is given below. Use the mid-point rule with $n = 4$ to estimate the value of the definite integral $\int_0^2 f(x)\,dx$.



(a) 3

(b) 2

(c) -3

(d) -1

(e) -2

22.(5 pts.) The volume of a **spherical** balloon is growing at a constant rate of 1 inch per second. How fast is the radius r growing when $r = 2$ inches? (Note that the volume of a sphere of radius r is $V = \frac{4\pi}{3}r^3$.)

(a) $\frac{1}{4\pi}$ inch/sec (b) $\frac{1}{16\pi}$ inch/sec (c) $(4\pi - 1)$ inch/sec

(d) $\frac{1}{8\pi}$ inch/sec (e) $(8\pi - 1)$ inch/sec

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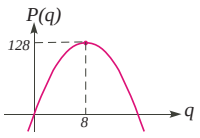
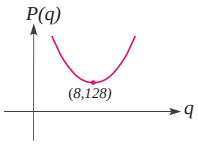
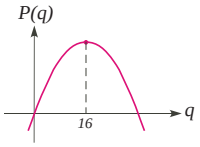
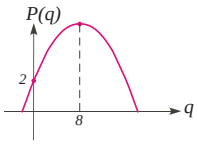
23.(5 pts.) The price function for an item is given by $p = -2q + 36$, where q is the number of items and p is the price in dollars. If the cost function is given by $C(q) = 4q$, which of the following is the graph of the **profit** function $P(q)$?

(a) 

(b) 

(c) 

(d) 

(e) None of these choices.

24.(5 pts.) The GDP of a country at the beginning of 2006 was \$500 billion dollars and it was growing at a rate of \$20 per year. If the GDP of the country t years after 2006 is $G(t)$ (in billions of dollars), write down a linear function that you would use to estimate the GDP near the beginning of 2006.

(a) $G(t) \approx -20t + 500$ (b) $G(t) \approx 500t + 20$ (c) $G(t) \approx 20t + 500$

(d) $G(t) \approx -500t + 20$ (e) $G(t) \approx 25t + 20$

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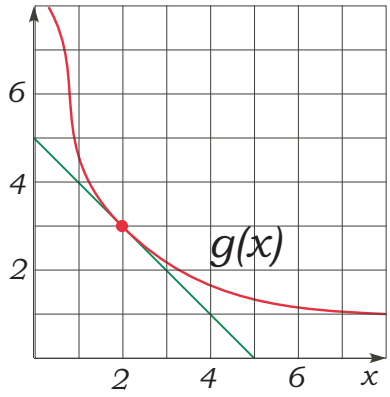
25.(5 pts.) The graph of $g(x)$ and its tangent line at $x = 2$ are given below. Find the instantaneous rate of change of $f(x) = \dfrac{g(x)}{2x+1}$ at $x = 2$.



(a) $1/25$

(b) $11/25$

(c) $-1/25$

(d) $-1/2$

(e) $-11/25$

26.(5 pts.) Referring to the same graph of $g(x)$ as above, find the derivative of the function $k(x) = \ln(g(x))$ at $x = 2$.

(a) None of these choices. (b) $-1/3$ (c) $1/3$

(d) $1/2$ (e) $-1/2$

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27.(5 pts.) The area between the curves $y = x^2$ and $y = x$, for $0 \leq x \leq 2$, is given by the following expression:

(a) $\displaystyle\int_0^1 (x - x^2)\,dx + \int_1^2 (x - x^2)\,dx$

(b) $\displaystyle\int_0^1 (x - x^2)\,dx - \int_1^2 (x^2 - x)\,dx$

(c) $\displaystyle\int_0^2 (x^2 - x)\,dx$

(d) $\displaystyle\int_0^2 (x - x^2)\,dx$

(e) $\displaystyle\int_0^1 (x - x^2)\,dx + \int_1^2 (x^2 - x)\,dx$

28.(5 pts.) Find the value of A such that $F(x) = A(3x - 1)^5$ is an antiderivative of $f(x) = 2(3x - 1)^4$.

(a) $\dfrac{6}{5}$ (b) 36 (c) $\dfrac{2}{15}$

(d) 12 (e) $\dfrac{2}{5}$

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29.(5 pts.) What are the global maximum and global minimum values of the function $f(x) = x^3 + 3x^2 - 9x + 1$ for x in $[0, 2]$?

(a) The global maximum value is 3, the global minimum is 0.

(b) The global maximum value is 5, the global minimum is 1.

(c) The global maximum value is 3, the global minimum is 1.

(d) The global maximum value is 1, the global minimum is -4.

(e) The global maximum value is 3, the global minimum is -4.

30.(5 pts.) Find the limit $\lim_{h \to 0} \dfrac{\ln(2(x+h)+3) - \ln(2x+3)}{h}$.

(a) $\dfrac{3}{2x+3}$

(b) $\dfrac{1}{2x+3}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{2}{2x+3}$

(e) $\dfrac{1}{2}$

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Math 10250, Final Exam - Version A Instructor: ANSWERS
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