Math 10250 Exam 1 Solutions – Fall 2006

1. We have $\dfrac{\Delta q}{\Delta p} = \dfrac{5,000}{-50} = -100$. Therefore using the point-slope formula for the equation of the (demand) line having slope -100 and passing through (200, 25000) gives $q - 25,000 = -100(p - 200)$, or $q = -100p + 45,000$.

2. Using the properties of the logarithms and the given, we have $\log_3\left(\dfrac{9}{a}\right) = \log_3 9 - \log_3 a = 2 - 5 = -3$.

3. The balance is $3{,}000(1 + \frac{0.08}{4})^{4\cdot 3} = 3{,}000(1.02)^{12}$.

4. We have $\displaystyle\lim_{x\to 5} \frac{x^2 - 3x - 10}{x - 5} = \lim_{x\to 5} \frac{(x-5)(x+2)}{x-5} = \lim_{x\to 5}(x+2) = 5 + 2 = 7$.

5. Since $\ln(x+3)$ is defined only when $x + 3 > 0$ we must have $x > -3$.

6. Since $-2 < 1 < 3$ and $2 > 1 > 0$ by the intermediate value theorem for continuous functions we conclude that $f(x) = 1$ has a solution in the intervals $[-1, 0]$ and $[1, 2]$.

7. If $y(t)$ is the amount of this substance at time t (in days) then $y(t) = 80e^{-kt}$. Since $40 = y(5) = 80e^{-k\cdot 5}$ we have $\dfrac{1}{2} = e^{-5k}$, or $-\ln 2 = -5k$, or $k = (\ln 2)/5$.

8. Solving $y = \dfrac{2}{x+1}$ for x gives $xy + y = 2$, or $xy = 2 - y$, or $x = \dfrac{2-y}{y}$. Switching the variables x and y gives the inverse function $g(x) = \dfrac{2-x}{x}$.

9. Since the vertex of the parabola is at $(2, -1)$ we have that $f(x)$ must be of the form $f(x) = a(x-2)^2 - 1$. Since $5 = f(0) = a(0-2)^2 - 1 = 4a - 1$ we have $4a = 6$ or $a = 3/2$. Therefore $f(x) = 1.5(x-2)^2 - 1$.

10. $f(x)$ is not continuous at $x = 5$ since $\displaystyle\lim_{x\to 5} f(x) = 2$, which is not equal to $f(5) = -1$.

11. Since $5x^2 - 5 = 5(x-1)(x+1)$ and $x^2 - 3x + 2 = (x-1)(x-2)$ simplifying gives $f(x) = 5(x+1)/(x-2)$. Thus $x = 2$ (the zero denominator) is the only the vertical asymptote. Since $\displaystyle\lim_{x\to\pm\infty} \frac{5(x+1)}{x-2} = 5$ we have that $y = 5$ is the only horizontal asymptote.

12. Solving $4x + 2y = 6$ for y gives $2y = -4x + 6$ or $y = -2x + 3$. Thus the given line has slope -2. Using the point-slope formula gives $y - (-3) = -2(x-1)$ or $y + 3 = -2x + 2$ or $y = -2x - 1$.

13. (i) The cost function is: $C(x) = 50x + 2{,}500{,}000$

    (ii) The revenue function is: $R(x) = x \cdot p = x(-0.05x + 900)$

    (iii) The profit function is: $P(x) = R(x) - C(x) = -0,05x^2 + 850x - 2{,}500{,}000$.

14. We have: $f(x) = -3x^2 + 24x + 2 = -3(x^2 - 8x) + 2 = -3(x^2 - 2\cdot 4\cdot x) + 2 = -3(x^2 - 2\cdot 4\cdot x + 4^2 - 4^2) + 2 = -3[(x-4)^2x - 4^2] + 2 = -3(x-4)^2 + 50$.

15. (a) We have $\displaystyle\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2 - 1}{x - 1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1}(x+1) = 2$.

    (b) The function $f(x)$ is not continuous at $x = 1$ since $f(1) = 3$, which is different from its limit there.

16. We have $\displaystyle\lim_{h\to 0} \frac{(3+h)^2 - 9}{h} = \lim_{h\to 0} \frac{9 + 6h + h^2 - 9}{h} = \lim_{h\to 0} \frac{6h + h^2}{h} = \lim_{h\to 0} \frac{h(6+h)}{h} = \lim_{h\to 0}(6+h) = 6$.

17. The number of whales in the distant future will be $\displaystyle\lim_{t\to\infty}\left(300 + \frac{800}{t+2}\right) = 300 + \lim_{t\to\infty} \frac{800}{t+2} = 300 + 0 = 300$.

18. Let P be an amount. Then, we must have $Pe^{r\cdot 10} = 2P$ or $e^{10r} = 2$ or $10r = \ln 2$ or $r = 0.1\ln 2 \approx 0.07$.

19. Plan (A) at 65, that is 35 years later, gives a balance of $100{,}000e^{0.08\cdot 35} = 100{,}000e^{2.8} \approx 1{,}644{,}460$, which is bigger than 1,500,000. Thus (A) is the preferred plan.

20. The amount is equal to: $1{,}000\left(1 + \dfrac{0.05}{365}\right)^{100} \approx 1013.79$.