Math 10250 Name:____________________

Exam 2 Instructor:____________________

October 24, 2006 Section:____________________

Calculators are allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 1 hour and 15 minutes to do the test. You may leave earlier if you are finished.

Part I consists of 10 multiple choice questions worth 5 points each. Record your answers by placing an $\times$ through one letter for each problem on **this** answer sheet.

Part II consists of 5 pages of partial credit problems worth a total of 50 points. Write your answer and show **all** your work on the page on which the question appears.

You are taking this exam under the honor code.

GOOD LUCK

1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e
9. a b c d e
10. a b c d e

For grading use:

Q1- Q10	
Pg. 6	
Pg. 7	
Pg. 8	
Pg. 9	
Pg. 10	
Total	

Part I: Multiple Choice Questions (5 Points Each)

1. The demand curve $p = D(q)$ for a certain iPod model is displayed in Figure 1. The price p is in dollars and the quantity q is in millions of units. Find the rate of change of p with respect to q, that is $\dfrac{dp}{dq}$, (in dollars per million units) when $q = 20$ million units.

   (**a**) 1

   (**b**) -1

   (**c**) 10

   (**d**) -10

   (**e**) 150

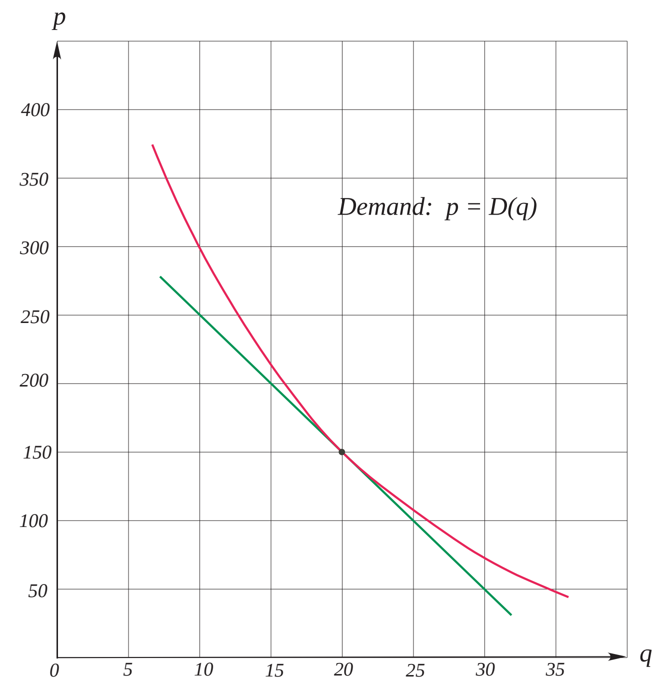

Figure 1

2. Find $\lim\limits_{h\to 0} \dfrac{(8+h)^{2/3} - 8^{2/3}}{h}$. (Hint: Think derivative!)

   (**a**) 1/3

   (**b**) 8/3

   (**c**) 4

   (**d**) 0

   (**e**) The limit does not exist.

3. The *unemployment rate* at the end of September was 4.6 % and was **falling** at the **rate** of 0.1% per month. Assuming that the unemployment rate is a differentiable function of time, $U = U(t)$, write a formula that predicts its approximate value in the near future. Count time in months and let $t = 0$ be the end of September. (Hint: Use linear approximation.)

(**a**) $U(t) \approx 0.1 - 4.6t$

(**b**) $U(t) \approx 0.1 + 4.6t$

(**c**) $U(t) \approx 4.6 - 0.1t$

(**d**) $U(t) \approx 4.6 + 0.1t$

(**e**) $U(t) \approx 4.6t - \dfrac{0.1t^2}{2}$

4. The height of a ball thrown upwards from the ground with initial velocity 64 feet per second is given by the formula $s(t) = -16t^2 + 64t$. Find the velocity of the ball at the moment it hits the ground.

(**a**) 16 feet per second

(**b**) 32 feet per second

(**c**) 64 feet per second

(**d**) -32 feet per second

(**e**) -64 feet per second

5. The profit of a furniture company making (and selling) x desks per month is given by $P(x) = -0.5x^2 + 200x - 5000$ dollars. Considering that the company is currently producing 185 desks per month, which of the following statements is **not** true?

(**a**) The marginal profit at $x = 185$ is positive.

(**b**) It is profitable for the company to increase production.

(**c**) The marginal profit at $x = 199$ is positive.

(**d**) The marginal profit at $x = 201$ is negative.

(**e**) The profit function is decreasing for all $x > 190$.

6. Suppose the demand for a certain product is given by $p = f(q)$, where p is the price per unit in dollars, and q is the number of units of the quantity sold. If $f(2000) = 500$ and $f'(2000) = -0.05$, find the **marginal revenue** when $q = 2000$.

(**a**) -100 dollars per unit

(**b**) -0.05 dollars per unit

(**c**) 200 dollars per unit

(**d**) 400 dollars per unit

(**e**) 500 dollars per unit

7. Assume that radioactive carbon decays according to the formula $y(t) = Ae^{-0.00018t}$, where A is the initial amount and t is the time in years. A wood sample is excavated from an ancient city, and is found to have a rate of radioactive carbon decay of 4 disintegrations per minute (dpm). On the other hand, a similar fresh wood sample is found to have a decay rate of 6 dpm. Determine the age of the excavated sample. (Hint: If T is the age of the sample, the information in this problem gives you the ratio $y'(T)/y'(0) = 4/6$.)

(**a**) $\dfrac{-1}{0.00018} \ln \dfrac{2}{3}$

(**b**) $\dfrac{1}{0.00018} \ln \dfrac{2}{3}$

(**c**) $\dfrac{1}{0.00018}$

(**d**) $\dfrac{-1}{0.00018} \cdot \dfrac{2}{3}$

(**e**) $\dfrac{1}{0.00018} \cdot \dfrac{2}{3}$

8. A cup of tea, initially 400°F hot, is brought into a room whose temperature is kept at a steady 70°F. Five minutes later, the temperature of the tea is **decreasing** at a rate of 4°F per minute. Let $H(t)$ denote the temperature of the tea as a function of time. Which of the following statements is **not** true? (Hint: You do not need to find a formula for $H(t)$.)

(**a**) $H(0) = 400$

(**b**) $H'(5) = 4$

(**c**) $H(t)$ is decreasing

(**d**) $H'(t)$ is never positive

(**e**) $\lim_{t\to\infty} H(t) = 70$

9. If a pebble is dropped into a pond then it creates an expanding circular ripple. The radius and the area of the circle are both functions of time, and they are related to each other by the formula $A = \pi r^2$. Suppose that at a certain moment the radius is 50 centimeters and is increasing at a rate of 0.4 centimeters per second. How fast is the area increasing at that moment (in cm^2 per seconds)?

(**a**) 20π

(**b**) 40π

(**c**) 100π

(**d**) 1000π

(**e**) 2500π

10. For the function $f(x)$ whose graph is shown in Figure 2, which of the following statements is **not** true?

(**a**) $f(x)$ is *not* differentiable at $x = 1, 2$ and 3.

(**b**) $f(x)$ is *not* continuous at $x = 3$.

(**c**) $f'(x)$ is positive for $x < 0$.

(**d**) $f'(x)$ is negative for $x > 3$.

(**e**) $x = 1$ is the only critical point.

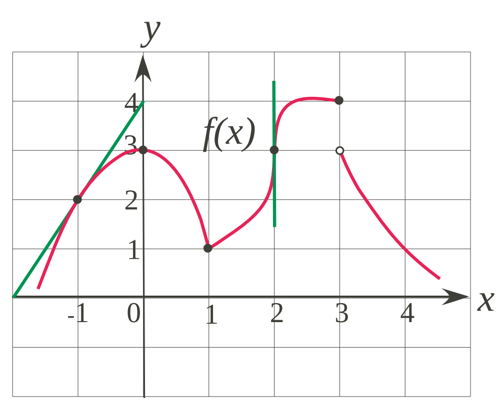

Figure 2

Part II: Partial Credit Questions (10 Points per Page)

Show all work and put your final answer in the space provided. **No credit** will be given for a correct answer without showing how it was obtained. You will receive no credit if the answer is not in the space provided.

11. **(a)** Using the **definition of the derivative** as the limit of average rates of change, find the derivative of the function $f(x) = 5x^2$.

Answer: ____________________

(b) If $f(x) = e^{2x} + \ln x$, then find $f'''(x)$.

Answer: ____________________

12. If $g(x)$ is a differentiable function such that $g(1) = 3$ and $g'(1) = 2$ then find the exact value of each of the following derivatives at $x = 1$:

(a) If $Q(x) = \dfrac{g(x)}{x+1}$ then $Q'(1) \stackrel{?}{=}$ ____________

(b) If $L(x) = \ln(g(x))$ then $L'(1) \stackrel{?}{=}$ ____________

(c) If $E(x) = e^{g(x)}$ then $E'(1) \stackrel{?}{=}$ ____________

13. **(a)** Find the linear approximation of $f(x) = \ln x$ at $x = 1$.

Answer: ____________________

(b) The position of a particle moving along the x-axis is given by

$$x(t) = 5t^2 + t^{5/2},$$

where t is in seconds and x is in meters. Find its acceleration.

Answer: ____________________

14. **(a)** By using implicit differentiation, find an expression for $\dfrac{dy}{dx}$, where

$$4x^2 + xy = 3y^3 + 4.$$

Answer: ____________________

(b) For the same curve as above, find the **slope** of its tangent line at the point $(1, 0)$.

Answer: ____________________

15. **(a)** Find all critical points to the function $f(x) = x^3 - 9x^2 + 24x - 3$.

Answer: ____________________

(b) Assume that apple production, in millions of bushels per month, in the states of Indiana and Michigan this harvesting season is given by

$$A(t) = 5te^{-0.5t}, \quad 0 \le t \le 4,$$

where t is the time in months since the beginning of August. Find the time at which production is maximum. Provide a **full** explanation by using the **first derivative test**. (Hint: Look at $A'(t) = 5(1 - 0.5t)e^{-0.5t}$.)

Answer: ____________________