Math 10250 Exam 2 Solutions – Fall 2006

1. The answer is (d). Indeed, $\dfrac{dp}{dq} = \dfrac{100-200}{25-15} = \dfrac{-100}{10} = -10.$

2. The answer is (a). Indeed, $\lim\limits_{h\to 0} \dfrac{(x+h)^{\frac{2}{3}} - x^{\frac{2}{3}}}{h} = (x^{\frac{2}{3}})' = \dfrac{2}{3}x^{-\frac{1}{3}}$. Hence the desired limit is the value of $(x^{\frac{2}{3}})'$ at $x=8$, i.e., $\frac{2}{3}8^{-\frac{1}{3}} = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$.

3. The answer is (c). Indeed, $U(0) = 4.6$ and $U^{'}(0) = -0.1$. Hence, for t close to 0, we have $U(t) \simeq U(0) + U^{'}(0)(t-0) = 4.6 - 0.1t$.

4. The answer is (e). Indeed, the ball hits the ground at the positive value of t such that $s(t) = 0$. $s(t) = -16t^2 + 64t = 0$ means $t(-16t+64) = 0$, i.e., since $t > 0$, $t = \frac{64}{16} = 4$ (seconds). Since $v(t) = s'(t) = -32t + 64$, we have $v(4) = -32\cdot 4 + 64 = -64$ (feet per second).

5. The answer is (e). Indeed, $MP(x) = P^{'}(x) = -x + 200$. Thus, (a), (c) and (d) are seen to be true statements by plugging in the respective values of x. (b) holds true since (a) does. Finally, (e) is false since for $190 < x < 200$, we have $P^{'}(x) > 0$, that is P is increasing in that interval.

6. The answer is (d). Indeed, $R(q) = p\cdot q = f(q)\cdot q$. Hence $MR(q) = R^{'}(q) = f^{'}(q)\cdot q + f(q)$. Thus (in dollars per unit), $MR(2000) = f^{'}(2000)\cdot 2000 + f(2000) = -0.05\cdot 2000 + 500 = -100 + 500 = 400$.

7. The answer is (a). Indeed, we want to find the value of T, and we know that $y'(T) = 4$ and $y'(0) = 6$. Since $y'(T) = -0.00018Ae^{-0.00018T}$ and $y'(0) = -0.00018A$, we have $\dfrac{y'(T)}{y'(0)} = e^{-0.00018T} = \dfrac{4}{6} = \dfrac{2}{3}$. By taking the natural logarithm of both sides, it follows that $-0.00018T = \ln\frac{2}{3}$, i.e., $T = \dfrac{-1}{0.00018}\ln\dfrac{2}{3} \approx 2252.58$ years.

8. The answer is (b). Indeed, the information given tells us that $H(0) = 400$, i.e. that (a) is true. Moreover, the temperature of the tea will clearly always decrease and eventually approach that of the room. This means that all of (c), (d) and (e) hold true. (b) is false since $H^{'}(5) = -4$ (not 4).

9. The answer is (b). Differentiating both sides of the equation $A(t) = \pi r^2(t)$ with respect to t gives $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$. Using the information given we obtain $\dfrac{dA}{dt} = 2\pi\cdot 50\cdot 0.4 = 40\pi$.

10. The answer is (e). Indeed, (a) is true since at $x=1$ the graph has a "corner", at $x=2$ it has a vertical slope, and $x=3$ is a point of discontinuity. The latter observation implies that (b) is also true. (d) holds since f is decreasing for $x>3$, and, similarly, (c) holds since f is increasing for $x<0$. (e) is false since the critical points of f are $x = 0, 1$ and 2. Note, f is not differentiable at $x = 1, 2$.

11. (a) $f^{'}(x) = \lim\limits_{h\to 0}\dfrac{5(x+h)^2 - 5x^2}{h} = \lim\limits_{h\to 0}\dfrac{5x^2 + 10xh + 5h^2 - 5x^2}{h} = \lim\limits_{h\to 0}\dfrac{5h(2x+h)}{h} = \lim\limits_{h\to 0}5(2x+h) = 10x.$

(b) We have $f^{'}(x) = 2e^{2x} + \dfrac{1}{x}$, $f^{''}(x) = 4e^{2x} - \dfrac{1}{x^2}$ and $f^{'''}(x) = 8e^{2x} + \dfrac{2}{x^3}$.

12. (a) $Q'(x) = \dfrac{g'(x)(x+1) - g(x)\cdot 1}{(x+1)^2}$. Hence $Q'(1) = \dfrac{2\cdot(1+1) - 3\cdot 1}{(1+1)^2} = \dfrac{1}{4}$.

(b) $L'(x) = \dfrac{g'(x)}{g(x)}$. Hence $L'(1) = \dfrac{g'(1)}{g(1)} = \dfrac{2}{3}$. (c) $E'(x) = e^{g(x)}g'(x)$. Hence $E'(1) = e^{g(1)}g'(1) = e^3\cdot 2 = 2e^3$.

13. (a) Since $f'(x) = \dfrac{1}{x}$, the desired approximation is $f(x) \simeq f(1) + f'(1)(x-1) = \ln 1 + \dfrac{1}{1}(x-1) = x - 1$.

(b) $a(t) = x''(t) = (10t + \dfrac{5}{2}t^{\frac{3}{2}})' = 10 + \dfrac{5}{2}\cdot\dfrac{3}{2}t^{\frac{1}{2}} = 10 + \dfrac{15}{4}\sqrt{t}$ (in meters per squared second).

14. (a) Differentiating both sides of the equation gives $\dfrac{d}{dx}(4x^2 + xy) = \dfrac{d}{dx}(3y^3 + 4)$, or $8x + (y + x\dfrac{dy}{dx}) = 9y^2\dfrac{dy}{dx} + 0$. Solving for $\dfrac{dy}{dx}$ gives $\dfrac{dy}{dx} = \dfrac{-8x - y}{x - 9y^2}$.

b) The desired slope m is the value of $\dfrac{dy}{dx}$ at $(1,0)$. Hence, (a) gives $m = \dfrac{-8\cdot 1 - 0}{1 - 9\cdot 0^2} = \dfrac{-8}{1} = -8$.

15. (a) $f^{'}(x) = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x-2)(x-4)$. Hence the critical points are $x = 2, 4$.

(b) $A'(t) = 5(1 - 0.5t)e^{-0.5t}$. Since $5e^{-0.5t} > 0$ for all t, we have $A'(t) = 0$ for $1 - 0.5t = 0$, i.e., for $t = \frac{1}{0.5} = 2$. Also, $A(t)$ increases for $1 - 0.5t > 0$, i.e., for $t < 2$, and $A(t)$ decreases for $1 - 0.5t < 0$, i.e., for $t > 2$. It follows that when $t = 2$, i.e., at the beginning of October, the desired production is maximum.