# Math 10250 Exam 3 Solutions — Fall 2006

1. $R(q) = \frac{72q}{q+2}$, $q \geq 0$; $R'(q) = \frac{144}{(q+2)^2} > 0$, $R''(q) = \frac{-288}{(q+3)^3} < 0$; the answer is (c).

2. $P(q) = \frac{72q}{q+2} - 4q$, $q \geq 0$; $P'(q) = \frac{144}{(q+2)^2} - 4$ and $P$ is maximum at $q = 4$ with $P(4) = 32$; the answer is (a).

3. $s(0)$ is the minimum of the position function; the answer is (d).

4. $f'(x) < 0$ on $(-\infty, -1)$ and $f'(x) > 0$ on $(-1, +\infty)$; the answer is (b).

5. Since $f''(x) = (3x^2 + 1)e^{x^3+x} > 0$ the answer is (b).

6. $P(x) = -0.01x^2 + 10x - 1000$, $P'(x) = -0.002x + 10$ so $P$ has a maximum value at $x = 500$ with $P(500) = 1500$; the answer is (d).

7. The function to be minimized is time (as a function of $x$); the answer is (a).

8. The intensity from point B is $I_B(x) = \frac{0.2}{10-x}$; the answer is (c).

9. $y(t) = 2e^{t/2} + t^2 + C$ and $5 = y(0) = 2 + C$ so that $C = 3$ and $y(2) = 2e + 7$; the answer is (b).

10. $P(t) = \int 20te^{-t^2}dt = -10e^{-t^2} + C$ and $4 = P(1) = -\frac{10}{e} + C$ so that $C = 4 + \frac{10}{e}$ and $P(8) = -10e^{-64} + 4 + \frac{10}{e}$; the answer is (a).

11. (a) $f'(x) = 15x^4 - 20x^3$ and $f''(x) = 60x^2(x-1)$ so that $f''(0) = 0$ hence the test is inconclusive and $f''(4/3) > 0$ hence $x = 4/3$ is a local minimum.

    (b) Since $f'(x) = 2x + e^{2x}$ then $f(x) = \int(2x + e^{2x})\,dx = x^2 + \frac{1}{2}e^{2x} + C$. To find the constant we observe that $0 = f(0) = \frac{1}{2} + C$ so that $C = -\frac{1}{2}$ and hence $f(x) = x^2 + \frac{1}{2}e^{2x} - \frac{1}{2}$.

12. Let $x$ denote the width and $y$ the height of the page. Then $xy = 200$ and the printed area is $A(x) = (x-4)(\frac{200}{x} - 2) = \frac{2(x-4)(100-x)}{x}$ with $4 \leq x \leq 100$. Since $A'(x) = \frac{200}{x} - 2 - (x-4)\frac{200}{x^2} = \frac{-2(x-20)(x-20)(x+20)}{x^2}$

    and $A(20) > 0$ the desired dimensions are $x = 20$ and $y = 10$.

13. (a) $\int(e^x + e^{-x})\,dx = e^x - e^{-x} + C$.

    (b) If $u = x^2 + 1$ then $du = 2xdx$ and $\int \frac{2x}{x^2+1}dx = \int \frac{1}{u}\,du = \ln(x^2+1) + C$.

14. We have $C(x) = 10x + \frac{6250}{x+1} + 259$ where $x \geq 0$ and $C'(x) = 10 - \frac{6250}{(x+1)^2}$ with $x = 24$ the critical point. Since $C(0) = 6509$, $C(24) = 749$ and $\lim_{x\to\infty} C(x) = +\infty$ the answer is $x = 24$.

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