Name ______________________________ Date ___________________

Math 10250 Review for Exam 2

1. Use the definition of the derivative to find the derivative of each of the following functions.

   (a) $f(x) = -3x^2 + 4$ (b) $f(x) = \dfrac{1}{3x+8}$

2. Evaluate exactly (a) $\displaystyle\lim_{h\to 0} \frac{(7+h)^{10} - 7^{10}}{h}$ and (b) $\displaystyle\lim_{h\to 0} \frac{10^{7+h} - 10^7}{h}$ (Hint: Think derivative!)

   (Ans: (a) $10(7)^9$; (b) $10^7 \ln 10$)

3. The demand curve of a certain product is shown in Figure 1. The price p is measured in dollars and the quantity q in millions of units.

   (a) Find the marginal revenue MR at $q = 20$. (Ans: -20)

   (b) Use linear approximation to estimate $R(20.1)$. (Ans: 198)

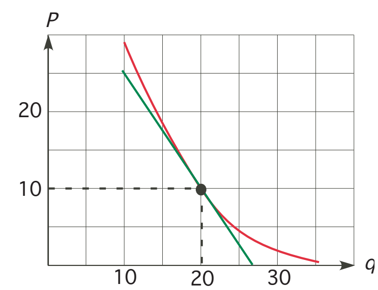


Figure 1

4. A ball is thrown into the air and its height in feet (measured from the ground) after t seconds is given by $s = -16t^2 + 32t + 48$ until it hits the ground.

   (a) What is the initial height of the ball? (Ans: 48 ft)

   (b) What is its velocity at the end of 1, and 1.5 seconds? In what direction (up or down) is it moving at the end of 1 and 1.5 second? (Ans: $v(t) = -32t + 32$)

   (c) At what time does the ball hit the ground? (Ans: 3 sec.)

   (d) What is the ball's acceleration at the end of 0.5 seconds? What is the ball's acceleration after 1 second? (Ans: -32 ft/s^2)

5. $\left(x^4 - e^{3x}\right)''' \stackrel{?}{=}$ (Ans: $24x - 27e^{3x}$)

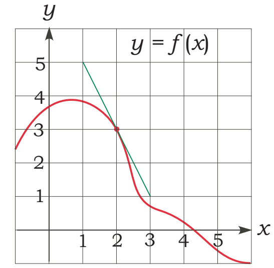
6. The demand for an item is $p = 80 - 0.2x$ and its cost function is $C(x) = 20x + 100$, where x is the quantity of the item. Find the marginal revenue, cost and profit. If every item made is sold, should the company increase production to increase profit when $x = 100$? when $x = 200$? Explain.

   (Ans: $R'(x) = 80 - 0.4x$, $C'(x) = 20$, $P'(x) = R'(x) - C'(x)$)

7. Let $f(x) = x^3 g(2/x)$. If $g(1) = 3$ and $g'(1) = 10$, then find $f'(2)$. (Ans: -4)

8. The GDP of a country at the beginning of 2006 was \$500 billion dollars and it was growing at a rate of \$20 per year. Use tangent line approximation to estimate the GDP of this country at the end of the third quarter. (Hint: Let $G(t)$ be the GDP. Then $G(0) = 500$ and $G'(0) = 20$. Thus $G(3/4) \approx \ldots$)

9. Given the graph of $f(x)$, find each of the following derivatives below.



(a) If $p(x) = f(x) \cdot x^3$ then $p'(2) \stackrel{?}{=}$

(b) If $q(x) = \dfrac{f(x)}{f(x)+1}$ then $q'(2) \stackrel{?}{=}$

(c) If $r(x) = \ln(f(x)+e)$ then $r'(2) \stackrel{?}{=}$

(d) $s(x) = e^{f(x)} + (f(x))^3$ then $s'(2) \stackrel{?}{=}$

Ans: (a) 20, (b) $-1/8$, (c) $\dfrac{-2}{3+e}$, (d) $-2e^3 - 54$

10. If \$3,000 is deposited in an in an account paying 6% annual interest, compounded **continuously**. How long it will take for the balance to reach \$6,000. Ans. $t = \frac{\ln 2}{0.06}$

11. How much money must you invest in an account paying 3% annual interest compounded **continuously** in order to have a balance of \$20,000 in 10 years?

(Ans. $20000e^{-0.3}$)

12. Use the approximation $\log_2 3 \approx 1.585$ and $\log_2 5 \approx 2.322$ to approximate the following:

(a) $\log_2 30 \stackrel{?}{\approx}$ Ans.4.907

(b) $\log_2 15 \stackrel{?}{\approx}$ Ans.3.907

(c) $\log_2(9/10) \stackrel{?}{\approx}$ Ans.-0.152

13. A chain of gourmet food stores sells a delicacy prepared from a rare fish species. Suppose that the amount of delicacy available at any time during the 16-week season is given by

$$w = 1000te^{-0.02t^2}, \qquad 0 \le t \le 16,$$

where w is the number of pounds and t is the time in weeks. Suppose the price per pound is $p = 500 - 0.08w$. How fast (in dollars per week) is the **revenue** from this delicacy changing at the end of 8 weeks? (Ans: $-62,506.99$ dollars/week)

14. Suppose a rectangular tank, whose base is a square of length 5 feet, is filling with water at the rate of 0.5 cubic feet per minute. How fast is the water level rising? (Ans: 1/50 ft/min)

15. You have just brought your Starbucks coffee into your room, which is kept at the temperature of 70°F. Five minutes later the temperature of the coffee is 350°F and is decreasing at a rate of 7°F per minute. Write a differential equation modeling the temperature $H(t)$ of your coffee. (a) Find $H(5)$ and $H'(5)$. (b) Is $H'(5)$ positive or negative? What does this say? (c) Finally, find a formula for $H(t)$. (Ans: $H(t) = 70 + 280e^{-0.025(t-5)}$)

16. The radioactive carbon in a piece of wood taken from an ancient cave decays at the rate of 6 disintegrations per minute (dpm), while the radioactive carbon in a similar sample of fresh wood decays at the rate of 8 dpm. Using 5,568 years as the half-life of radioactive carbon, estimate the age of the wood. (Ans: Age$\approx$ 2310.93 years)

17. What are the different names of the derivative?