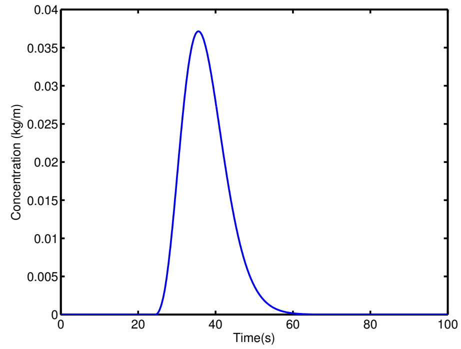

Figure 1: Breakthrough Curve for Problem 2.

Problem 1. Mobile Immobile Models

Consider the standard mobile immobile model, that is

$$\frac{\partial C_1}{\partial t} + u\frac{\partial C_1}{\partial x} = -\alpha C_1 + \alpha C_2 \tag{1}$$

$$\beta\frac{\partial C_2}{\partial t} = \alpha C_1 - \alpha C_2 \tag{2}$$

but with slightly different initial conditions than we have considered in class

$$C_1(t=0) = 0 \qquad C_2(t=0) = \delta(x) \tag{3}$$

(i) Laplace transform the governing equations

(ii) Solve the Laplace transform equations and find a solution for C_1 the mobile concentration. Can you invert it into real time space?

(iii) Produce breakthrough curves for the case where $u = 1$ at a distance $x = 1$ for all possible combinations of $\alpha = 1$ or 10 and $\beta = 1$ or 10 and discuss the results and how the values of α and β change you BTCs. Discuss these results and explain if they align with your physical intuition of what α and β represent.

(iv) Compare your results to the case we studied in class where the initial condition was $C_1(t) = \delta(x)$ and $C_2(t=0) = 0$. Explain similarities and differences, in particular considering the specific influence of α and γ parameters.

Problem 2. Conservative MIM Model and Real Data

You are provided with the data depicted in figure 1 corresponding to a breakthrough curve in a stream where you know that the mean velocity $u = 2m/s$ and that transport is advection dominated so dispersion effects are negligible. You measure breakthrough curves at a downstream distance of $x = 50m$. This dataset corresponds to a conservative tracer and

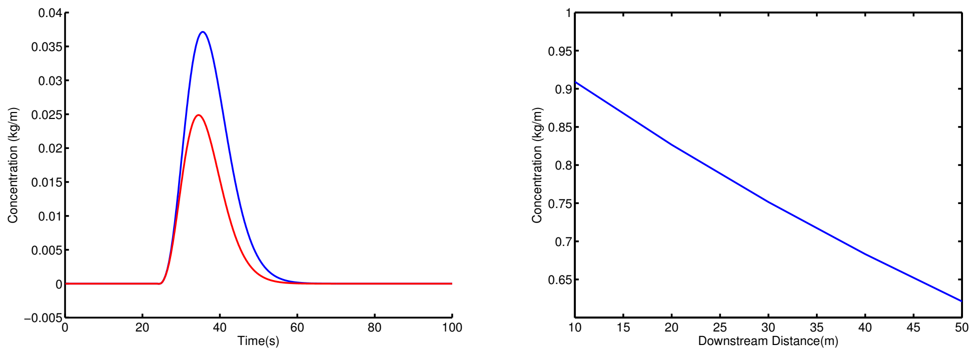

Figure 2: Breakthrough Curve for Problem 2.

you set up an initial condition such that all the mass is thrown in as a pulse in the mobile section or the stream. Assume that a MIM model is the appropriate model to use and based on these results estimate the exchange parameter α and capacity coefficient β.

You can do this by raw brute force by simply tweaking the parameters until they match your observation reasonably and this is a standard way to do this, but try to think and see if you can be clever about this too. The 'clever' part I am thinking of (and please be cleverer than me if you can) is not something we discussed in class explicitly, but as a hint think about when the peak of the curve arrives and what this might tell you about at least one of the two parameters.The data for the BTC is in the file HW72Conservative.mat (download from course website)

Problem 3. Reactive Transport MIM

For the same stream you perform two reactive transport experiments, one with a pulse injection and the other where you run the system to steady state and maintain a constant concentration at $x = 0$, while measuring concentrations at $x = 10$, 20, 30, 40 and 50 m. The data is shown in figure 2. For the particular solute you have used you know that it will not degrade in the open channel, and only react in the immobile region. From the given data calculate the reaction rate in the immobile region as well as the effective rate you would get if you ignored mobile-immobile exchange. Are these consistent for the pulse and steady state cases? The data for the BTC is the in the file HW72Reactive.mat and the steady state measurements in HW72ReactiveSS.mat

Problem 4. Reactive Transport MIM

The last problem we worked on in class was the steady state reactive system where reactions only happen in the immobile zone. Solve the same problem, that is calculate the steady state distribution of C_1, but now where reactions only happen in the mobile zone at a rate γ. What changes and why? Can you think of a real system where this might apply? Finally perform the calculation for a case where reaction in the mobile zone happens at a rate γ_1 and in the immobile region at a rate γ_2. Again, comment on the result.