Sample Problem 1 - Dimensional Homogeneity

$$\mathcal{F} = K_a v t^{-1} + K_b \ell t^{-1} + K_c t^{-2}$$

$$[\mathcal{F}] = M L T^{-2}$$
$$[v] = L T^{-1}$$
$$[t] = T$$
$$[\ell] = L$$

$$\Rightarrow \quad [K_a v t^{-1}] = [K_a] L T^{-1} T^{-1} = [K_a] L T^{-2} = M L T^{-2}$$

$$[K_b \ell t^{-1}] = [K_b] L T^{-1} = [K_b] L T^{-1} = M L T^{-2}$$

$$[K_c t^{-2}] = [K_c] T^{-2} = [K_c] T^{-2} = M L T^{-2}$$

$$\therefore \quad [K_a] = M$$

$$[K_b] = M T^{-1}$$

$$[K_c] = M L$$

Solution

Sample Problem - Mass/Density

$(1.2\ kg\ m^{-3})$

Information : $V_T = 1\ m^3$ $\quad SG_1 = 2 \Rightarrow \rho_1 = 2000\ kg/m^3$ $\quad \rho_2 = 800\ kg/m^3$

$V_1 + V_2 = 1$

$\Rightarrow V_2 = 1 - V_1$ $\quad m_2 = 3m_1 \Rightarrow \rho_2 V_2 = 3\rho_1 V_1$

$$\therefore 3\rho_1 V_1 = \rho_2 (1 - V_1)$$

$$\Rightarrow V_1 = \frac{\rho_2}{3\rho_1 + \rho_2} = \frac{800}{6000 + 800} = 0.117\ m^3$$

$$V_2 = 0.883\ m^3$$

$V_{\rho 2} = 0.883$ $\quad V_{\rho 1} = 0.117$

$m_1 = \rho_1 V_1 = (2000)(0.117) = 234\ kg$ $\quad (3m_1 = 702\ kg)$

$m_2 = \rho_2 V_2 = (800)(0.883) = 706\ kg$

$m_1 = 234\ kg$ $\quad m_2 = 706\ kg$

Ideal Gas

Sample Problem $\Rightarrow \quad \rho = \frac{P}{RT}$

$T = 27°C$ or $300\,°K \quad \Rightarrow \quad 80°F$ OR $540°R$

$R_{air} = 1.716 \times 10^3 \ ft\ lb / slug\ °R$ OR $2.87 \times 10^2\ J/kg\ K$

$P = 14$ psi (assume gauge unless specified)

$\therefore P_{abs} = 14 + 14.7 = 28.7$ psi OR 198 kPa

Convert psi $\rightarrow lb\ ft^{-2} \Rightarrow 28.7\ (12)^2 = 4132.8\ lb\ ft^{-2}$

(SI) $\therefore \rho = \frac{(198 \times 10^3\ N m^{-2})}{(2.87 \times 10^2\ J\ kg^{-1} K^{-1})(300 K)} = 2.3\ N J^{-1} m^{-2} kg$

$[N] = kg\ m\ s^{-2}$
$[J] = kg\ m^2 s^{-2}$ [Energy Work = Force x distance]

$\rho = \boxed{2.3\ kg\ m^{-3}}$ GOOD

(British) $\therefore \rho = \frac{4132.8\ lb\ ft^{-2}}{1.716 \times 10^3\ ft\ lb\ slug^{-1} R^{-1}\ 540 R^{-1}} = \boxed{4.46 \times 10^{-3}}\ slug\ ft^{-3}$

@ 30000 ft $\Rightarrow T \approx -50°F \Rightarrow 410°R$ OR $-50°C \Rightarrow 220°K$

$P_{abs} = 4.3$ psi $\quad 26000\ N m^{-2}$

$\Rightarrow \rho \approx \frac{26000}{(287)(220)} = 0.41\ kg\ m^{-3}$

Sample Problem - Viscosity

$$v(y) = -1.1y^3 + 12y^2 + 13.2y \qquad 0 \le y \le 10$$

$$\tau = \mu \frac{dv}{dy} \qquad \text{where } \mu_{air} \approx 10^{-3}\ Ns\,m^{-2}$$

$$\therefore \ \tau_{max} \text{ at } \left.\frac{dv}{dy}\right|_{max}$$

$$\frac{dv}{dy} = -3.3y^2 + 24y + 13.2$$

$$\frac{d^2v}{dy^2} = -6.6y + 24 \Rightarrow \text{max, min of } \frac{dv}{dy} \text{ at } \frac{d^2v}{dy^2} = 0$$

$$\Rightarrow y = 24/6.6$$

$$\frac{d^3v}{dy^3} = -6.6 < 0 \Rightarrow \text{maximum}$$

Other possible max, min at boundary ($y = 0$ and $y = 10$)

$$\therefore \ \tau = (10^{-3})(-3.3y^2 + 24y + 13.2)$$

$$@\ y = 0 \Rightarrow \tau = 13.2 \times 10^{-3}\ N\,m^{-2}$$

$$@\ y = \frac{24}{6.6} \Rightarrow \tau = 56.8 \times 10^{-3}\ N\,m^{-2}$$

$$@\ y = 10 \Rightarrow \tau = -76.8 \times 10^{-3}\ N\,m^{-2}$$

Now question becomes are you concerned with actual τ or $|\tau|$ (magnitude)

If $\tau \Rightarrow$ min at $y = 10$ and max @ $y = 24/6.6$

If $|\tau| \Rightarrow$ min at $y = 0$ and max @ $y = 10$

Sample Problem

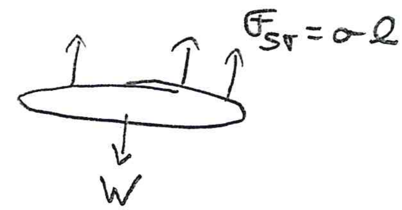
Speed of Sound - c=sqrt(E/rho)

$E_{V_{H_2O}} = 2.2 \times 10^9 \; Pa$ $\qquad$ $E_{V_{air}} = 10^5$

$\rho_{H_2O} = 1000$ $\qquad$ $\rho = 1$

$c_{H_2O} = 1.4 \times 10^3 \; m s^{-1}$ $\qquad$ $c_{air} \approx 316 \; m s^{-1}$

Sample Problem - Water Strider



For Equilibrium $\Sigma F = 0$

(i) $W = \sigma l$

$$l = W/\sigma = \frac{10^{-4}\ N}{7.3\times10^{-2}\ Nm^{-1}} = 1.36\times10^{-3}\ m = 1.36\ mm$$

(ii) $$l = \frac{750}{7.3\times10^{-2}} = 1\times10^{4}\ m = 10\ km\ (\approx 6\ miles)$$

End Chapter 1

Demos : ① Fire Piston
② Non Newtonian Fluid