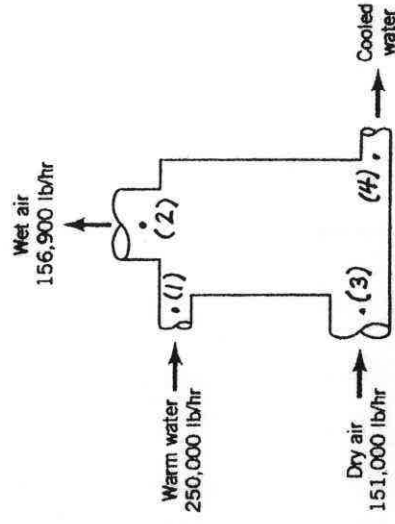


## 5.11

5.11 An evaporative cooling tower (see Fig. P5.11) is used to cool water from 110 to 80°F. Water enters the tower at a rate of 250,000 lb/hr. Dry air (no water vapor) flows into the tower at a rate of 151,000 lb/hr. If the rate of wet air flow out of the tower is 156,900 lb/hr, determine the rate of water evaporation in lb/hr and the rate of cooled water flow in lb/hr.



■ FIGURE P5.11

For steady flow of dry air

$$\dot{m}_3 = \dot{m}_2, \text{ dry air} \quad (1)$$

For steady flow of water

$$\dot{m}_1 = \dot{m}_2, \text{ water} + \dot{m}_4 \quad (2)$$

Also

$$\dot{m}_2 = \dot{m}_3, \text{ dry air} + \dot{m}_2, \text{ water} \quad (3)$$

Combining Eqs. 1 and 3 we get

$$\dot{m}_2, \text{ water} = \dot{m}_2 - \dot{m}_3 = \text{rate of water evaporation}$$

$$\text{So with weight} = \text{acceleration of gravity} \times \text{mass we have}$$

$$g\dot{m}_2, \text{ water} = 156,900 \frac{\text{lb}}{\text{hr}} - 151,000 \frac{\text{lb}}{\text{hr}} = \underline{\underline{5900 \frac{\text{lb}}{\text{hr}}}}$$

From Eq. 2 we get

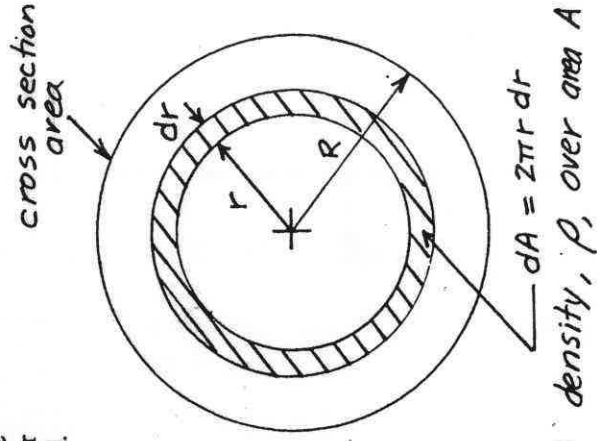
$$g\dot{m}_4 = g\dot{m}_1 - g\dot{m}_2, \text{ water} = \text{rate of cooled water flow}$$

$$\text{or } g\dot{m}_4 = 250,000 \frac{\text{lb}}{\text{hr}} - 5900 \frac{\text{lb}}{\text{hr}} = \underline{\underline{244,000 \frac{\text{lb}}{\text{hr}}}}$$

5.15 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c \left( \frac{R-r}{R} \right)^{1/n} \hat{\mathbf{i}}$$

where  $u_c$  = centerline velocity,  $r$  = local radius,  $R$  = pipe radius, and  $\hat{\mathbf{i}}$  = unit vector along pipe centerline. Determine the ratio of average velocity,  $\bar{u}$ , to centerline velocity,  $u_c$ , for (a)  $n = 5$ , (b)  $n = 6$ , (c)  $n = 7$ , (d)  $n = 8$ , (e)  $n = 9$ , (f)  $n = 10$ .



For any cross section area

$$\dot{m} = \rho A \bar{u} = \int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

Also

$$\mathbf{V} \cdot \hat{\mathbf{n}} = \mathbf{V} \cdot \hat{\mathbf{i}} = u_c \left( \frac{R-r}{R} \right)^{\frac{1}{n}}$$

Thus for a uniformly distributed density,  $\rho$ , over area  $A$

$$\bar{u} = \frac{\int_0^R u_c \left( \frac{R-r}{R} \right)^{\frac{1}{n}} 2\pi r dr}{\pi R^2}$$

and

$$\frac{\bar{u}}{u_c} = \frac{2 \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{2n^2 + 3n + 1} = \frac{2n^2}{2n^2 + 3n + 1}$$

$n$	$\frac{\bar{u}}{u_c}$
5	0.758
6	0.791
7	0.817
8	0.837
9	0.853
10	0.866

5.17

5.17 Two rivers merge to form a larger river as shown in Fig. P5.17. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown and the depth is 6 ft. Determine the value of  $V$ .

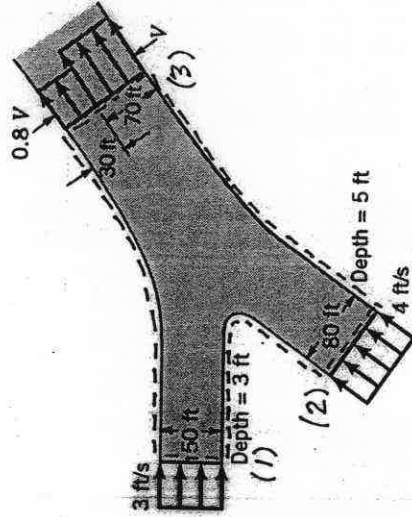


FIGURE P5.17

Use the control volume shown within broken lines in the sketch above. We note that  $\dot{m} = \rho A V$  and from the conservation of mass principle we get

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0.8V} + \dot{m}_V$$

Thus

$$\rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0.8V} 0.8V + \rho A_V V$$

and

$$V = \frac{A_1 V_1 + A_2 V_2}{A_{0.8V} + A_V} = \frac{(50\text{ft})(3\text{ft})(3\frac{\text{ft}}{\text{s}}) + (80\text{ft})(5\text{ft})(4\frac{\text{ft}}{\text{s}})}{(30\text{ft})(6\text{ft})(0.8) + (70\text{ft})(6\text{ft})}$$

$$V = \underline{\underline{3.63 \frac{\text{ft}}{\text{s}}}}$$

5.2.3

5.2.3 Determine the anchoring force required to hold in place the conical nozzle attached to the end of the laboratory sink faucet shown in Fig. P5.2.3 when the water flowrate is 10 gal/min. The nozzle weight is 0.2 lb. The nozzle inlet and exit inside diameters are 0.6 and 0.2 in., respectively. The nozzle axis is vertical and the axial distance between sections (1) and (2) is 1.2 in. The pressure at section (1) is 68 psi.

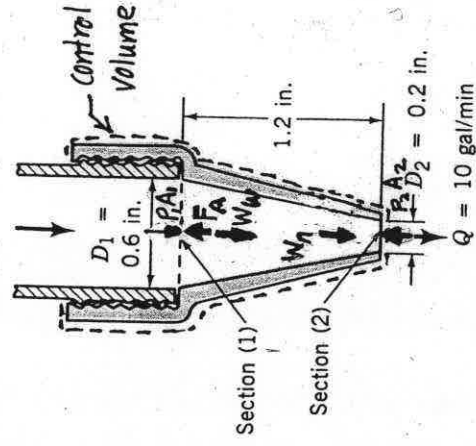


FIGURE P5.2.3

The analysis leading to the solution of this problem is similar to the one outlined in Example 5.6. Included in the control volume are the nozzle and the water in the nozzle at an instant. Application of the vertical or z-direction component of the linear momentum equation (Eq. 5.17) to the flow through this control volume leads to

$$F_A = \rho w_1^2 A_1 - \rho w_2^2 A_2 + W_n + P_1 A_1 + W_w - P_2 A_2 \quad (1)$$

which is Eq. 4 of Example 5.6.

The conservation of mass equation yields

$$\dot{m} = \rho w_1 A_1 = \rho w_2 A_2$$

thus Eq. 1 becomes

$$F_A = \dot{m} (w_1 - w_2) + W_n + P_1 A_1 + W_w - P_2 A_2 \quad (2)$$

The different terms in Eq. 2 are calculated below.

$$\dot{m} = \rho Q = (1.94 \frac{\text{slugs}}{\text{ft}^3}) (10 \frac{\text{gal}}{\text{min}}) (\frac{7.48 \text{ gal}}{\text{ft}^3}) (\frac{1}{60 \frac{\text{s}}{\text{min}}}) = 0.0432 \frac{\text{slug}}{\text{s}}$$

$$w_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(10 \frac{\text{gal}}{\text{min}}) (\frac{12 \text{ in.}}{\text{ft}})^2}{\pi (0.6 \text{ in.})^2} (\frac{7.48 \frac{\text{gal}}{\text{ft}^3}}{60 \frac{\text{s}}{\text{min}}}) = 11.4 \frac{\text{ft}}{\text{s}}$$

$$w_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{(10 \frac{\text{gal}}{\text{min}}) (\frac{12 \text{ in.}}{\text{ft}})^2}{\pi (0.2 \text{ in.})^2} (\frac{7.48 \frac{\text{gal}}{\text{ft}^3}}{60 \frac{\text{s}}{\text{min}}}) = 102 \frac{\text{ft}}{\text{s}}$$

$$P_1 A_1 = P_1 \frac{\pi D_1^2}{4} = (68 \frac{\text{lb}}{\text{in.}^2}) \pi (0.6 \text{ in.})^2 / 4 = 19.2 \text{ lb}$$

(cont)

5.23

(con't)

$$W_w = \rho g V_w = \rho g \frac{\pi}{12} (D_1^2 + D_2^2 + D_1 D_2) h$$

$$W_w = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \left(32.2 \frac{\text{ft}}{\text{s}^2}\right) \left(1 \frac{\text{lb}}{\text{ft}^2}\right) \left(1 \frac{\text{slug}}{\text{s}^2}\right) \frac{\pi}{12} \left[ (0.6 \text{ in.})^2 + (0.2 \text{ in.})^2 + (0.6 \text{ in.})(0.2 \text{ in.}) \right] \left(\frac{1.2 \text{ in.}}{12 \text{ in./ft}}\right)$$

$$W_w = 0.00591 \text{ lb}$$

$$P_2 A_2 = P_2 \pi \frac{D_2^2}{4} = \left(0 \frac{\text{lb}}{\text{in.}^2}\right) \pi \frac{(0.2 \text{ in.})^2}{4} = 0 \text{ lb}$$

Thus with Eq. 2

$$F_A = \left(0.0432 \frac{\text{slug}}{\text{s}}\right) \left(11.4 \frac{\text{ft}}{\text{s}} - 102 \frac{\text{ft}}{\text{s}}\right) \left(1 \frac{\text{lb}}{\text{slug}} \frac{\text{ft}}{\text{s}^2}\right) + 0.2 \text{ lb} + 19.2 \text{ lb} + 0.00591 \text{ lb} = 0 \text{ lb}$$

$$\underline{\underline{F_A = 15.5 \text{ lb}}}$$

5.27

5.27 A converging elbow (see Fig. P5.2.7) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m³ between sections (1) and (2). The water volume flowrate is 0.4 m³/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place.

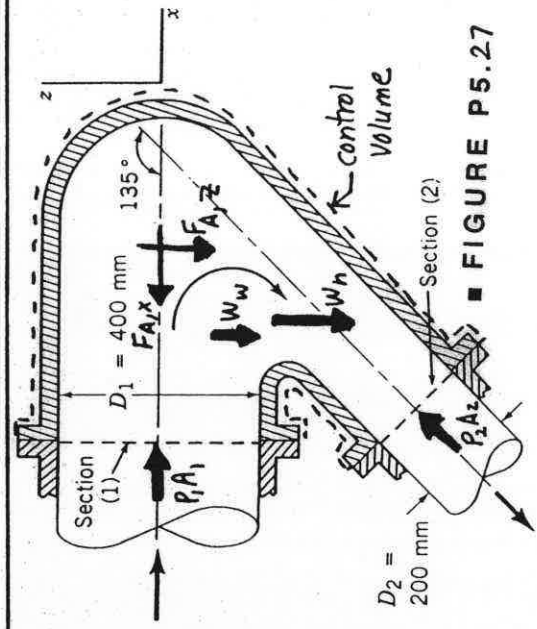


FIGURE P5.2.7

A control volume that contains the elbow and the water within the elbow between sections (1) and (2) as shown in the sketch above is used. Application of the horizontal or x direction component of the linear momentum equation yields (Eq. 5.17)

$$-u, \rho u, A_1 - V_2 \cos 45^\circ \rho V_2 A_2 = P_1 A_1 - F_{A,x} + P_2 A_2 \cos 45^\circ$$

From conservation of mass

$$\dot{m} = \rho u_1 A_1 = \rho V_2 A_2 = \rho Q \tag{1}$$

Thus

$$F_{A,x} = \frac{\rho Q^2}{A_1} + \frac{\rho Q^2 \cos 45^\circ}{A_2} + P_1 A_1 + P_2 A_2 \cos 45^\circ = \frac{\rho Q^2}{\pi D_1^2} + \frac{\rho Q^2 \cos 45^\circ}{\pi D_2^2} + P_1 \frac{\pi D_1^2}{4} + P_2 \frac{\pi D_2^2 \cos 45^\circ}{4}$$

$$F_{A,x} = \left( 999 \frac{\text{kg}}{\text{m}^3} \right) \left( 0.4 \frac{\text{m}^3}{\text{s}} \right)^2 \frac{1}{\pi} \left[ \frac{\left( \frac{1000 \text{ mm}}{\text{m}} \right)^2}{(400 \text{ mm})^2} + \frac{\cos 45^\circ \left( \frac{1000 \text{ mm}}{\text{m}} \right)^2}{(200 \text{ mm})^2} \right] \left( 1 \frac{\text{N}}{\text{kg} \frac{\text{m}}{\text{s}^2}} \right) + \frac{\pi \left( \frac{1000 \text{ N}}{\text{Pa} \cdot \text{m}^2} \right)^2}{4 \left( \frac{1000 \text{ mm}}{\text{m}} \right)^2} \left[ (150 \text{ kPa}) (400 \text{ mm})^2 + (90 \text{ kPa}) (200 \text{ mm})^2 \cos 45^\circ \right]$$

$$F_{A,x} = \underline{\underline{25,700 \text{ N}}}$$

Application of the vertical or z direction component of the linear momentum equation leads to

$$-V_2 \sin 45^\circ \rho V_2 A_2 = P_2 A_2 \sin 45^\circ - F_{A,z} - W_w - W_e$$

which when combined with Eq. 1 gives

$$F_{A,z} = \frac{\rho Q^2 \sin 45^\circ}{A_2} + P_2 A_2 \sin 45^\circ - W_w - W_e = \frac{\rho Q^2 \sin 45^\circ}{\pi \frac{D_2^2}{4}} + P_2 \frac{\pi D_2^2 \sin 45^\circ}{4} - \rho g V - m_e g \tag{CON't}$$

5.27

(con't)

$$F_{A,z} = \left( 999 \frac{\text{kg}}{\text{m}^3} \right) \left( 0.4 \text{ m} \right)^2 \sin 45^\circ \left( \frac{1 \text{ N}}{\text{kg} \cdot \text{m}} \right) + \frac{(90 \text{ kPa}) \pi (200 \text{ mm})^2 \sin 45^\circ}{4 \left( 1000 \frac{\text{mm}}{\text{m}} \right)^2}$$

$$= \left( 999 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) (0.2 \text{ m}^3) \left( \frac{1 \text{ N}}{\text{kg} \cdot \text{m}} \right) - (12 \text{ kg}) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) \left( \frac{1 \text{ N}}{\text{kg} \cdot \text{m}} \right)$$

$$F_{A,z} = \underline{\underline{3520 \text{ N}}}$$



5.32

5.32 The four devices shown in Fig. P5.32 rest on frictionless wheels, are restricted to move in the  $x$  direction only and are initially held stationary. The pressure at the inlets and outlets

we apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force  $F_A$ .

If  $F_A$  is in the direction shown in the sketches, motion will be to the left. If  $F_A$  is

in a direction opposite to that shown, the motion is to the right. If  $F_A = 0$ , there is no horizontal motion.

For sketch (a)

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F_A$$

Since  $F_A$  is to the left, motion is to the right.  
For sketch (b)

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since  $V_1 > V_2$ , then  $F_A$  is to the left and motion is to the right.  
For sketch (c) (note: flow is into CV at (1))

$$-V_1 \rho V_1 A_1 = F_A$$

and  $F_A$  is to the left so motion is to the right.

For sketch (d)

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = F_A$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

$$\text{and } V_1 < V_2$$

so  $F_A$  is to the right and motion is to the left.

of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.

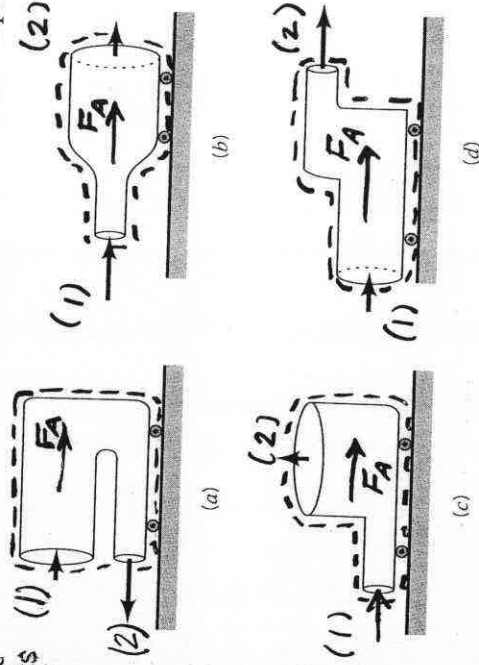


FIGURE P5.32



5.41

5.41 Two water jets of equal size and speed strike each other as shown in Fig. P5.41. Determine the speed,  $V$ , and direction,  $\theta$ , of the resulting combined jet. Gravity is negligible.

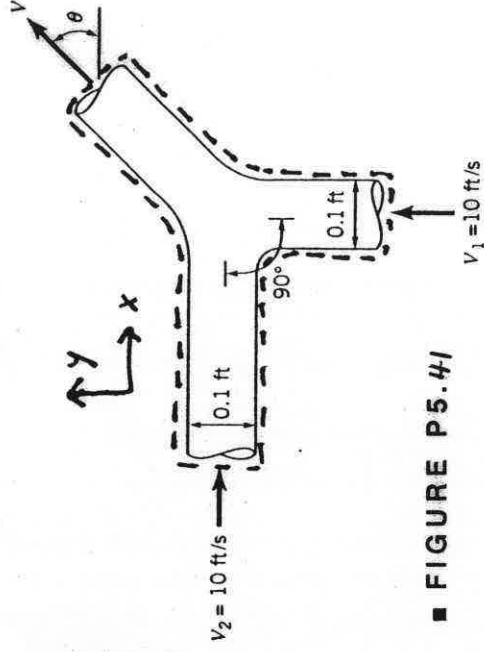


FIGURE P5.41

For the control volume shown in the sketch above the linear momentum equation for the  $x$  and  $y$  directions are, for the  $x$  direction

$$-V_2 \rho V_2 A_2 + (V \cos \theta) \rho V A = 0 \quad (1)$$

and for the  $y$  direction

$$-V_1 \rho V_1 A_1 + (V \sin \theta) \rho V A = 0 \quad (2)$$

Also for conservation of mass we have

$$\rho V_1 A_1 + \rho V_2 A_2 - \rho V A = 0$$

From Eqs. 1 and 2 we get

$$\frac{V_2^2 A_2}{V_1^2 A_1} = \frac{\cos \theta}{\sin \theta} = \cot \theta$$

so

$$\theta = \cot^{-1} \frac{V_2^2 A_2}{V_1^2 A_1} = \cot^{-1} \left[ \frac{(10 \frac{\text{ft}}{\text{s}})^2 \pi \frac{(0.1 \text{ft})^2}{4}}{(10 \frac{\text{ft}}{\text{s}})^2 \pi \frac{(0.1 \text{ft})^2}{4}} \right] = \underline{\underline{45^\circ}}$$

Now, combining Eqs. 2 and 3 we get

$$-V_1^2 A_1 + V \sin \theta (V_1 A_1 + V_2 A_2) = 0$$

or

$$V = \frac{V_1^2 A_1}{\sin \theta (V_1 A_1 + V_2 A_2)}$$

$$V = \frac{(10 \frac{\text{ft}}{\text{s}})^2 \pi \frac{(0.1 \text{ft})^2}{4}}{(\sin 45^\circ) \left[ (10 \frac{\text{ft}}{\text{s}}) \pi \frac{(0.1 \text{ft})^2}{4} + (10 \frac{\text{ft}}{\text{s}}) \pi \frac{(0.1 \text{ft})^2}{4} \right]}$$

and

$$V = \underline{\underline{7.07 \frac{\text{ft}}{\text{s}}}}$$

5.49

5.49 A sheet of water of uniform thickness ( $h = 0.01$  m) flows from the device shown in Fig. P5.49. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the  $y$  component of the anchoring force necessary to hold this device stationary.

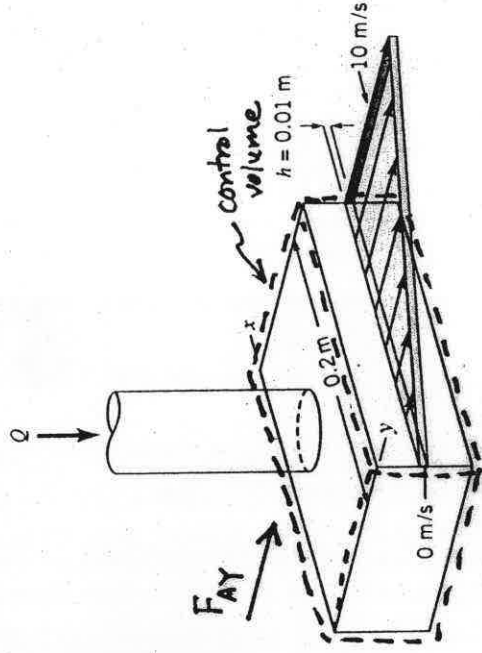


FIGURE P5.49

A control volume that contains the box portion of the device and the water in the box as shown in the sketch above is used. Application of the  $y$ -direction component of the linear momentum equation yields

$$F_{Ay} = \sum N_{y\text{slit}} \rho V_{\text{slit}} A_{\text{slit}} = \int_{x=0}^{0.2} \rho v^2 h dx$$

The variation of  $v$  with  $x$  is linear or

$$v = 50x \frac{\text{m}}{\text{s}}$$

Thus

$$F_{Ay} = \rho \int_0^{0.2} (50x)^2 h dx = \rho (50)^2 h \frac{x^3}{3} \Big|_0^{0.2}$$

or

$$F_{Ay} = \left(999 \frac{\text{kg}}{\text{m}^3}\right) \left(50 \frac{\text{m}}{\text{s}}\right)^2 (0.01 \text{ m}) \left(\frac{0.2 \text{ m}}{3}\right)^3 \left(1 \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right)$$

and

$$F_{Ay} = \underline{\underline{66.6 \text{ N}}}$$

5.53

5.53 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.53. The exit cross section area of each of the two nozzles is 0.04 in.<sup>2</sup> and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. (a) Determine the resisting torque required to hold the sprinkler head stationary. (b) Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. (c) Determine the angular velocity of the sprinkler if no resisting torque is applied.

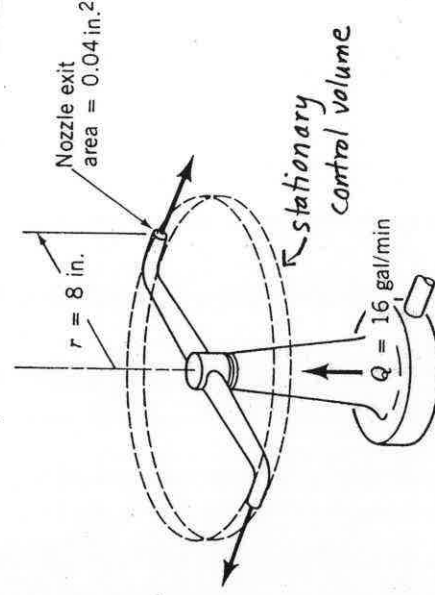


FIGURE P5.53

This is similar to Example 5.10.

(a) To determine the resisting torque required to hold the sprinkler head stationary we use the moment-of-momentum torque equation (Eq. 5.25). Thus,

$$T_{\text{shaft}} = m r_2 V_{\theta 2} = \rho Q r_2 V_{\theta 2} \quad (1)$$

For  $V_{\theta 2}$  we use

$$V_{\theta 2} = \frac{Q}{2A_{\text{nozzle exit}}} = \frac{(16 \frac{\text{gal}}{\text{min}})}{2(0.04 \text{ in.}^2)} (144 \frac{\text{in.}^2}{\text{ft}^2})$$

or

$$V_{\theta 2} = 64.17 \frac{\text{ft}}{\text{s}} = \frac{2(0.04 \text{ in.}^2)(7.48 \frac{\text{gal}}{\text{ft}^3})(60 \frac{\text{s}}{\text{min}})}{2(0.04 \text{ in.}^2)}$$

With Eq. 1 we obtain

$$T_{\text{shaft}} = \frac{(1.94 \frac{\text{slugs}}{\text{ft}^3})(16 \frac{\text{gal}}{\text{min}})(8 \text{ in.})(64.17 \frac{\text{ft}}{\text{s}})(1 \frac{\text{lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}})}{(7.48 \frac{\text{gal}}{\text{ft}^3})(60 \frac{\text{s}}{\text{min}})} (12 \frac{\text{in.}}{\text{ft}})$$

and

$$T_{\text{shaft}} = \underline{\underline{2.96 \text{ ft} \cdot \text{lb}}}$$

(b) To determine the resisting torque associated with a sprinkler speed of 500 rev/min we use Eq. 1 again. However, with rotation we have

$$V_{\theta 2} = W_2 - U_2 \quad (2)$$

For  $W_2$  we use

$$W_2 = \frac{Q}{2A_{\text{nozzle exit}}} = \frac{(16 \frac{\text{gal}}{\text{min}})(144 \frac{\text{in.}^2}{\text{ft}^2})}{(2)(0.04 \text{ in.}^2)(7.48 \frac{\text{gal}}{\text{ft}^3})(60 \frac{\text{s}}{\text{min}})} = 64.17 \frac{\text{ft}}{\text{s}}$$

5.53

(cont.)

For  $U_2$  we use

$$U_2 = r_2 \omega = \frac{(8 \text{ in.}) (500 \frac{\text{rev}}{\text{min}}) (2\pi \frac{\text{rad}}{\text{rev}})}{(12 \frac{\text{in.}}{\text{ft}}) (60 \frac{\text{s}}{\text{min}})} = 34.91 \frac{\text{ft}}{\text{s}}$$

Thus with Eq. 2 we have

$$V_{\theta, 2} = 64.17 \frac{\text{ft}}{\text{s}} - 34.91 \frac{\text{ft}}{\text{s}} = 29.26 \frac{\text{ft}}{\text{s}}$$

and with Eq. 1 we obtain

$$T_{\text{shaft}} = \frac{(1.94 \frac{\text{slug}}{\text{ft}^3}) (16 \frac{\text{ga}}{\text{min}}) (8 \text{ in.}) (29.26 \frac{\text{ft}}{\text{s}}) (1 \frac{\text{lb}}{\text{slug} \cdot \text{ft}^2})}{(748 \frac{\text{ga}}{\text{ft}^3}) (60 \frac{\text{s}}{\text{min}}) (12 \frac{\text{in.}}{\text{ft}})}$$

and

$$T_{\text{shaft}} = \underline{\underline{1.35 \text{ ft} \cdot \text{lb}}}$$

(c) To determine the angular velocity of the sprinkler if no resisting torque is applied we use the combination of Eqs. 1 and 2 to obtain

$$U_2 = W_2$$

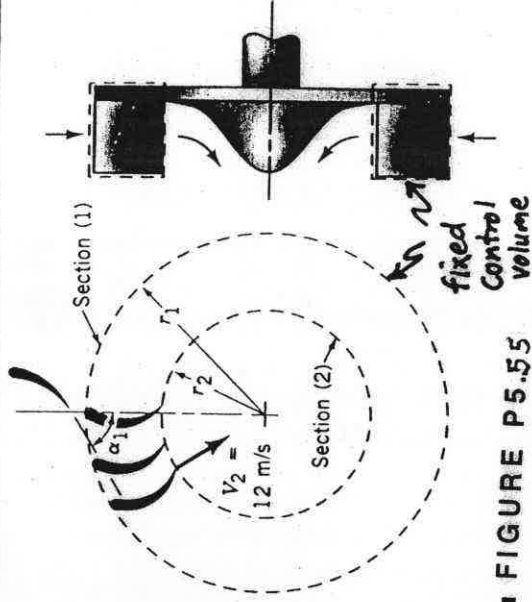
$$\text{or } \omega = \frac{W_2}{r_2} = \frac{(64.17 \frac{\text{ft}}{\text{s}}) (12 \frac{\text{in.}}{\text{ft}})}{(8 \text{ in.})} = 96.3 \frac{\text{rad}}{\text{s}}$$

The rotor speed,  $N$ , is thus

$$N = (96.3 \frac{\text{rad}}{\text{s}}) \frac{(60 \frac{\text{s}}{\text{min}})}{(2\pi \frac{\text{rad}}{\text{rev}})} = \underline{\underline{920 \frac{\text{rev}}{\text{min}}}}$$

5.5.5

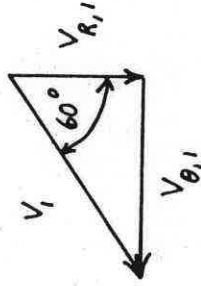
5.5.5 An inward flow radial turbine (see Fig. P5.55) involves a nozzle angle,  $\alpha_1$ , of  $60^\circ$  and an inlet rotor tip speed,  $U_1$ , of  $6 \text{ m/s}$ . The ratio of rotor inlet to outlet diameters is  $2.0$ . The absolute velocity leaving the rotor at section (2) is radial with a magnitude of  $12 \text{ m/s}$ . Determine the energy transfer per unit mass of fluid flowing through this turbine if the fluid is (a) air or (b) water.



■ FIGURE P5.55  
To determine the energy transfer per unit mass we use the moment-of-momentum energy equation (Eq. 5.33) to obtain

$$w_{\text{shaft net out}} = U_1 V_{\theta,1} \quad (1)$$

and we note that the result is independent of the fluid involved. The value of  $V_{\theta,1}$  can be ascertained with the help of the section (1) velocity triangle sketched below.



From the velocity triangle we note that

$$V_{\theta,1} = V_{r,1} \tan 60^\circ \quad (2)$$

With conservation of mass we obtain

$$V_{r,1} = V_{r,2} \frac{A_2}{A_1} = V_{r,2} \frac{r_2}{r_1} = (12 \frac{\text{m}}{\text{s}}) (\frac{1}{2}) = 6 \frac{\text{m}}{\text{s}}$$

Thus with Eq. 2 we obtain

$$V_{\theta,1} = (6 \frac{\text{m}}{\text{s}}) \tan 60^\circ = 10.4 \frac{\text{m}}{\text{s}}$$

and with Eq. 1 we get

$$w_{\text{shaft net out}} = (6 \frac{\text{m}}{\text{s}}) (10.4 \frac{\text{m}}{\text{s}}) \left( 1 \frac{\text{N}}{\text{kg} \cdot \frac{\text{m}}{\text{s}^2}} \right) = 62.4 \frac{\text{N} \cdot \text{m}}{\text{kg}}$$

for (a) air and (b) water.

5.67

5.67 Air flows past an object in a 2-m-diameter pipe and exits as a free jet as shown in Fig. P5.67. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m<sup>2</sup>, respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. (a) Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. (b) Determine the force that the air puts on the object.

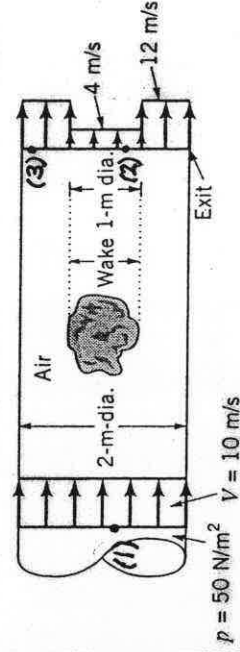


FIGURE P5.67

(a) For flow from (1) to (2) the energy equation

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 - h_L + h_s = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2 \text{ with } p_2 = 0, h_s = 0, \text{ and } z_1 = z_2 \text{ gives}$$

$$h_L = \frac{p_1}{\rho} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{50 \frac{N}{m^2}}{12 \frac{N}{m^3}} + \frac{(10 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} - \frac{(4 \frac{m}{s})^2}{2(9.81 \frac{m}{s^2})} = \underline{\underline{8.45 m}}$$

(b) The x-component of the momentum equation

$$\sum u_{out} \rho A_{out} V_{out} - \sum u_{in} \rho A_{in} V_{in} = \sum F_x$$

for this flow becomes

$$V_1 \rho (-V_1) A_1 + V_2 \rho (+V_2) A_2 + V_3 \rho (+V_3) A_3 = p_1 A_1 - R_x$$

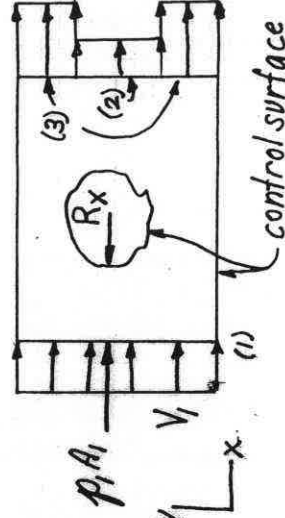
Thus,

$$R_x = p_1 A_1 + \rho V_1^2 A_1 - \rho V_2^2 A_2 - \rho V_3^2 A_3$$

$$= (50 \frac{N}{m^2}) \frac{\pi}{4} (2m)^2 + 1.23 \frac{kg}{m^3} [(10 \frac{m}{s})^2 \frac{\pi}{4} (2m)^2 - (4 \frac{m}{s})^2 \frac{\pi}{4} (1m)^2 - (12 \frac{m}{s})^2 \frac{\pi}{4} (2m)^2 - (1m)^2]$$

or

$$R_x = \underline{\underline{110 N}}$$





5.73

5.73 An incompressible liquid flows steadily along the pipe shown in Fig. P5.73. Determine the direction of flow and the head loss over the 6-m length of pipe.

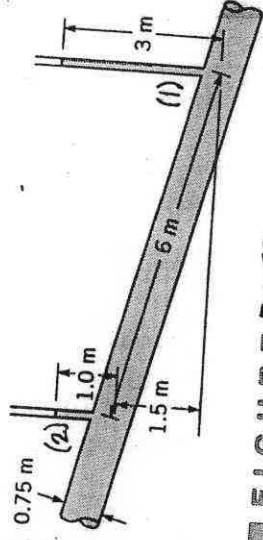


FIGURE P5.73

Assume flow from (1) to (2) and use the energy equation (Eq. 5.57) to get for the contents of the control volume shown:

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_L - h_s$$

Thus,

$$h_L = \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2 = 3\text{ m} - 1.0\text{ m} - 1.5\text{ m} = \underline{\underline{0.5\text{ m}}}$$

And since  $h_L > 0$ , the assumed direction of flow is correct.

The flow is uphill.



5.81 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.81 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.

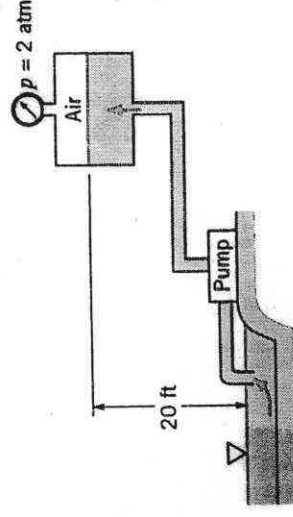


FIGURE P5.81

$$\frac{p_1}{\rho} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\rho} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = 0, z_1 = 0, V_1 = 0, \text{ and } z_2 = 20 \text{ ft.}$$

Thus,

$$(1) \quad h_s = h_L + \frac{p_2}{\rho} + z_2$$

Also,

$$Q = [(1000 \text{ gal}) / (10 \text{ min})] \left( \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 0.223 \frac{\text{ft}^3}{\text{s}}$$

so that

$$h_s = \frac{W_s}{\gamma Q} = \frac{(3 \text{ hp})(550 \frac{\text{ft} \cdot \text{lb}}{\text{hp} \cdot \text{s}})}{(62.4 \frac{\text{lb}}{\text{ft}^3})(0.223 \frac{\text{ft}^3}{\text{s}})} = 119 \text{ ft}$$

$$(a) \quad \text{If } p_2 = 2 \text{ atm} = 2(14.7 \frac{\text{lb}}{\text{in}^2}) (144 \text{ in}^2/\text{ft}^2) = 4,230 \frac{\text{lb}}{\text{ft}^2}, \text{ then from Eq. (1)}$$

$$h_s = h_L + \frac{4,230 \frac{\text{lb}}{\text{ft}^2}}{(62.4 \frac{\text{lb}}{\text{ft}^3})} + 20 \text{ ft} = h_L + 87.8 \text{ ft}$$

Thus, if

$$h_L \leq h_s - 87.8 \text{ ft} = 119 \text{ ft} - 87.8 \text{ ft} = 31.2 \text{ ft} \quad \underline{\underline{\text{the given pump will work for } p_2 = 2 \text{ atm.}}}$$

$$(b) \quad \text{If } p_2 = 3 \text{ atm} = 6,350 \frac{\text{lb}}{\text{ft}^2}, \text{ then}$$

$$h_s = h_L + \frac{6,350 \frac{\text{lb}}{\text{ft}^2}}{(62.4 \frac{\text{lb}}{\text{ft}^3})} + 20 \text{ ft} = h_L + 122 \text{ ft}$$

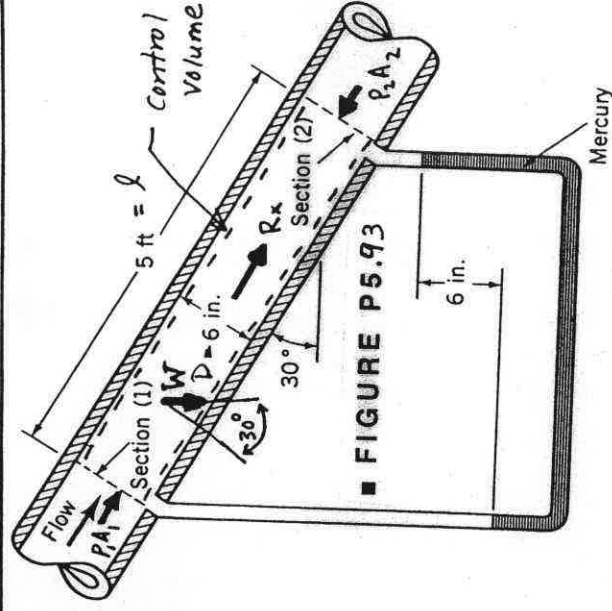
Thus, if this pump is to work

$$119 \text{ ft} = h_L + 122 \text{ ft}, \text{ or } h_L \leq -3 \text{ ft}$$

Since it is not possible to have  $h_L < 0$ , the pump will not work for  $p_2 = 3 \text{ atm}$ .

## 5.93

5.93 Water flows steadily down the inclined pipe as indicated in Fig. P5.93. Determine the following: (a) the difference in pressure  $p_1 - p_2$ , (b) the loss between sections (1) and (2), and (c) the net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).



(a) The difference in pressure,  $P_1 - P_2$ , may be obtained from the manometer (see Section 2.6) with the fluid statics equation

$$P_1 - P_2 = -\delta_{H_2O} \left[ (5 \text{ ft}) \sin 30^\circ + \left( \frac{6 \text{ in.}}{12 \frac{\text{in.}}{\text{ft}}} \right) \right] + \delta_{Hg} \left( \frac{6 \text{ in.}}{12 \frac{\text{in.}}{\text{ft}}} \right)$$

or

$$P_1 - P_2 = -\left( 62.4 \frac{\text{lb}}{\text{ft}^3} \right) \left[ (5 \text{ ft}) \sin 30^\circ + (0.5 \text{ ft}) \right] + (13.6) \left( 62.4 \frac{\text{lb}}{\text{ft}^3} \right) (0.5 \text{ ft}) = 237 \frac{\text{lb}}{\text{ft}^2}$$

and

$$P_1 - P_2 = 237 \frac{\text{lb}}{\text{ft}^2} \left( \frac{1}{144 \frac{\text{in.}^2}{\text{ft}^2}} \right) = \underline{\underline{1.65 \text{ psi}}}$$

(b) The loss per unit mass between sections (1) and (2) may be obtained with Eq. 5.56. Thus

$$\text{loss} = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = \left( 237 \frac{\text{lb}}{\text{ft}^2} \right) \left( \frac{1}{1.94 \frac{\text{slug}}{\text{ft}^3}} \right) + (3.22 \frac{\text{ft}}{\text{s}^2}) \left( 5 \text{ ft} \right) \left( \sin 30^\circ \right) \left( \frac{1 \text{ lb}}{\text{slug} \cdot \text{ft}} \right)$$

or

$$\text{loss} = \underline{\underline{203 \frac{\text{ft} \cdot \text{lb}}{\text{slug}}}} + (3.22 \frac{\text{ft}}{\text{s}^2}) \left( 5 \text{ ft} \right) \left( \sin 30^\circ \right) \left( \frac{1 \text{ lb}}{\text{slug} \cdot \text{ft}} \right)$$

(c) The net axial force exerted by the pipe wall on the flowing water may be obtained by using the axial component of the linear momentum equation (Eq. 5.17). Thus for the control volume shown above

$$R_x = -\frac{\pi D^2}{4} (P_1 - P_2) - \frac{\delta \pi D^2}{4} (l) \sin 30^\circ = -\frac{\pi D^2}{4} \left[ (P_1 - P_2) + \delta l \sin 30^\circ \right]$$

or

$$R_x = -\frac{\pi}{4} \left( \frac{6 \text{ in.}}{12 \frac{\text{in.}}{\text{ft}}} \right)^2 \left[ 237 \frac{\text{lb}}{\text{ft}^2} + \left( 62.4 \frac{\text{lb}}{\text{ft}^3} \right) (5 \text{ ft}) \sin 30^\circ \right]$$

and

$$R_x = \underline{\underline{-77.2 \text{ lb}}} = \underline{\underline{77.2 \text{ lb}}} \text{ opposite to flow direction.}$$