3.8 The Bernoulli equation is valid for steady, inviscid, incompressible flows with constant acceleration of gravity. Consider flow on a planet where the acceleration of gravity varies with height so that \( g = g_0 - cz \), where \( g_0 \) and \( c \) are constants. Integrate \( F = \mathbf{m} \ddot{\mathbf{a}} \) along a streamline to obtain the equivalent of the Bernoulli equation for this flow.

\[
\sum \delta F_s = \delta m \mathbf{a}_s \quad \text{one obtains}
\]

\[
dp + \frac{1}{2} \rho d(V^2) + \gamma' dz \quad \text{where} \quad \gamma' = \gamma = \rho g
\]

(see Eq. 3.5)

Thus,

\[
dp + d\left(\frac{1}{2} \rho V^2 + \rho (g_0 - cz)\right) dz = 0, \quad \text{or by integrating from (i) to (a)}:
\]

\[
\int_{(i)}^{(a)} dp + \int_{(i)}^{(a)} d\left(\frac{1}{2} \rho V^2 + \rho (g_0 - cz)\right) dz = 0
\]

or

\[
\rho_{(a)} - \rho_{(i)} + \frac{1}{2} \rho (V_{(a)}^2 - V_{(i)}^2) + \rho g_0 (z_{(a)} - z_{(i)}) - \frac{1}{2} \rho c (z_{(a)}^2 - z_{(i)}^2) = 0
\]

Thus,

\[
\rho + \frac{1}{2} \rho V^2 + \rho g_0 z - \frac{1}{2} \rho c z^2 = \text{constant along a streamline.}
\]
3.16 A person holds her hand out of an open car window while the car drives through still air at 65 mph. Under standard atmospheric conditions, what is the maximum pressure on her hand? What would be the maximum pressure if the "car" were an Indy 500 racer traveling 200 mph?

\[
\frac{\rho_1}{g} \left( \frac{V_1}{2g} \right)^2 + Z_1 = \frac{\rho_2}{g} \left( \frac{V_2}{2g} \right)^2 + Z_2 \quad \text{with} \quad Z_1 = Z_2
\]

Thus,

\[
\rho_2 = \frac{\rho_1}{g} \left( \frac{V_1}{2g} \right)^2 = \frac{1}{2} \rho V_1^2 \quad \text{or} \quad A_2 = \frac{1}{2} \left( 2.38 \times 10^{-3} \text{ slug/ft}^2 \right) (95.3 \text{ ft/s})^2 = 10.8 \text{ in Hg}
\]

If \( V_1 = 200 \text{ mph} \left( \frac{88 \text{ ft/s}}{60 \text{ mph}} \right) = 293 \text{ ft/s} \), then

\[
A_2 = \frac{1}{2} \left( 2.38 \times 10^{-3} \text{ slug/ft}^2 \right) (293 \text{ ft/s})^2 = 102 \text{ in Hg}
\]
3.32
An inviscid, incompressible liquid flows steadily from the large pressurized tank shown in Fig. P3.32. The velocity of the liquid is 40 ft/s. Determine the specific gravity of the fluid in the tank.

\[ P_1 + z_1 + \frac{V_1^2}{2g} = P_2 + z_2 + \frac{V_2^2}{2g} + \rho g h \]

where \( P_1 = 10 \text{ psi} \), \( V_1 = 0 \), and \( z_1 = 0 \), and \( P_2 = 40 \text{ psi} \), \( V_2 = 15 \text{ ft/s} \), and \( z_2 = 11.5 \text{ ft} \).

\[ \frac{1440 \text{ lb/ft}^3}{2 (32.2 \text{ ft/s})^2} = \frac{40 \text{ psi}}{15 \text{ ft/s}^2} \]

This gives \( \rho = 44 \text{ lb/ft}^3 \).

\[ x = \frac{1440 \text{ lb/ft}^3}{62.4 \text{ lb/ft}^3} = 2.3 \]

Hence, \( \rho = \frac{1440 \text{ lb/ft}^3}{44 \text{ lb/ft}^3} = 32.3 \text{ lb/ft}^3 \)
At any location within the tube \( V = V_3 \) so that with \( V_1 = 0 \), \( \rho_1 = 0 \), and \( z_1 = 0 \),

\[
\frac{\rho_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{\rho_3}{\rho} + \frac{V_3^2}{2g} + z_3
\]

gives

\[
\frac{\rho_3}{\rho} = -z - \frac{V_3^2}{2g}
\]

Thus, the lowest pressure occurs at the point of maximum \( z \). That is, \( \rho_2 = -30 \, \text{kPa} \) and \( z_2 = 2 \, \text{m} \) so that

\[
-\frac{30 \times 10^3 \, \text{N/m}^2}{8.89 \times 10^3 \, \text{N/m}^2} = -2 \, \text{m} - \frac{V_3^2}{2(9.81 \, \text{m/s}^2)}
\]

or

\[
V_3 = 4.56 \, \text{m/s}
\]

but

\[
\frac{\rho_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{\rho_3}{\rho} + \frac{V_3^2}{2g} + z_3
\]

where \( z_3 = -(4-h) \) and \( \rho_3 = 0 \)

Thus,

\[
0 = \frac{(4.56 \, \text{m/s})^2}{2(9.81 \, \text{m/s}^2)} - (4-h)
\]

or

\[
h = 2.94 \, \text{m}
\]
3.58 If viscous effects are neglected and the tank is large, determine the flowrate from the tank shown in Fig. P3.58.

\[
\frac{P_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{g} + \frac{V_2^2}{2g} + z_2 \quad \text{where} \quad \rho_1 = \rho_2 + \xi h = \xi h
\]

\[z_1 = 0.7 \text{m}, \quad z_2 = 0, \quad \text{and} \quad v_1 = 0\]

Thus,

\[
\frac{\xi h}{g} + z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad V_2 = \sqrt{2g \left( \frac{\xi h}{g} + z_1 \right)} \quad \text{where} \quad \frac{\xi h}{g} = 0.81
\]

and

\[Q = A_2 V_2 = \frac{\pi}{4} \cdot 0.05 \cdot V_2 \]

Thus,

\[
Q = \frac{\pi}{4} (0.05 \text{m})^2 \sqrt{2(9.81 \text{m/s}^2)(0.81(2 \text{m}) + 0.7 \text{m})} = 0.0132 \text{ m}^3/\text{s}
\]
Determine the flowrate through the Venturi meter shown in Fig. P3.78 if ideal conditions exist.

\[ P_1 = 735 \text{ kPa}, \quad P_2 = 550 \text{ kPa} \]

\[ \gamma = 9.1 \text{ kJ/kg} \cdot \text{K} \]

\[ T = 31 \text{ mm} \]

\[ (2) 9.1 \text{ mm} \cdot \text{K} \]

**FIGURE P3.78**

\[ V = \frac{A_1 V_1}{A_2} \]

\[ V_1 = \frac{(D_1)^2}{2g} \]

\[ V_2 = \frac{(D_2)^2}{2g} \]

Therefore, \[ \frac{V_1^2}{2g} + Z_1 = \frac{V_2^2}{2g} + Z_2 \]

where \( Z_1 = Z_2 \) and \( A_1 V_1 = A_2 V_2 \)

\[ \frac{V_2}{V_1} = \frac{(D_2)^2}{(D_1)^2} \]

\[ V_2 = \sqrt{\left( \frac{2g}{(D_1)^2} \right) \left( Z_2 - \frac{D_1^2}{D_2^2} \gamma \right)} \]

\[ V_2 = \sqrt{\left( \frac{2g}{(D_1)^2} \right) \left( Z_2 - \frac{D_1^2}{D_2^2} \gamma \right)} \]

\[ Q = \frac{A_2 V_2}{\gamma} = \frac{\pi D_2^2 V_2}{\gamma} \]

\[ Q = \frac{\pi (0.045 \text{ m})^2 (21.5 \text{ m/s})}{6.10 \times 10^{-3} \text{ m}^3} = 6.10 \times 10^{-3} \text{ m}^3/\text{s} \]