S.5.2 Five liters per second of water enter the rotor shown in Video V5.1 and Fig. P5.52 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm$^2$. How large is the resisting torque required to hold the rotor stationary if (a) $\theta = 0^\circ$, (b) $\theta = 30^\circ$, and (c) $\theta = 60^\circ$?

To determine the torque required to hold the rotor stationary, we use the moment of momentum torque equation (Eq. 5.25) to obtain

$$ T_{\text{shaft}} = m V_{\text{out}} \times \cos \theta $$

(1)

We note that $m = \rho Q$

(2)

and $V_{\text{out}} = \frac{Q}{3A_{\text{nozzle, exit}}}$

(3)

Combining Eqs. 1, 2 and 3 we get

$$ T_{\text{shaft}} = \frac{\rho Q^2 V_{\text{out}} \cos \theta}{3 A_{\text{nozzle, exit}}} $$

(4)

(a) For $\theta = 0^\circ$ we use Eq. (4) to get

$$ T_{\text{shaft}} = \frac{(999 \text{ kg/m}^3)(5 \text{ liters})^2(0.5m)(1000 \text{ mm/m})^2(1 \text{ N/kg} \cdot \text{m})}{(1000 \text{ liters/m}^3)(3)(18 \text{ mm}^2)} $$

or

$$ T_{\text{shaft}} = 231 \text{ N.m} $$

(b) For $\theta = 30^\circ$ we use Eq. (4) to get

$$ T_{\text{shaft}} = \frac{(999 \text{ kg/m}^3)(5 \text{ liters})^2(0.5m)(\cos 30^\circ)(1000 \text{ mm/m})^2(1 \text{ N/kg} \cdot \text{m})}{(1000 \text{ liters/m}^3)(3)(18 \text{ mm}^2)} $$

or

$$ T_{\text{shaft}} = 200 \text{ N.m} $$

(c) For $\theta = 60^\circ$ we use Eq. (4) to get

$$ T_{\text{shaft}} = \frac{(999 \text{ kg/m}^3)(5 \text{ liters})^2(0.5m)(\cos 60^\circ)(1000 \text{ mm/m})^2(1 \text{ N/kg} \cdot \text{m})}{(1000 \text{ liters/m}^3)(3)(18 \text{ mm}^2)} $$

or

$$ T_{\text{shaft}} = 116 \text{ N.m} $$

5-54
S.5.4 Five liters per second of water enters the rotor shown in Video V5.10 and Fig. P5.54 along the axis of rotation. The cross-sectional area of each of the three nozzles exits normal to the relative velocity is 18 mm². How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = 0^\circ$, (b) $\theta = 30^\circ$, or (c) $\theta = 60^\circ$?

To determine the rotor angular velocity associated with zero shaft torque we can use the moment of momentum torque equation (Eq. 5.29),

$$T_{\text{shaft}} = m r_{\text{out}} v_{\text{out}} \cos \theta - v_{\text{out}}$$

(1)

We note that

$$v_{\text{out}} = r_{\text{out}} \omega$$

(2)

and

$$v_{\text{out}} = \frac{Q}{3A_{\text{exit nozzle}}}$$

(3)

Combining Eqs. 1, 2 and 3 we get

$$T_{\text{shaft}} = m r_{\text{out}} \left( \frac{Q \cos \theta}{3A_{\text{exit nozzle}}} - r_{\text{out}} \omega \right)$$

(4)

(a) For $\theta = 0^\circ$

From Eq. 4 we obtain for $T_{\text{shaft}} = 0$

$$\omega = \frac{Q \cos \theta}{3A_{\text{exit nozzle}}} = \frac{(5 \text{ liters}) (\cos 0^\circ)(1000 \text{ mm}^2/\text{m})^2}{3(18 \text{ mm}^2)(1000 \text{ liters/m}^2)(0.5 \text{ m})} = 185 \text{ rad/s}$$

(con't)
(b) For $\theta = 30^\circ$

From Eq. 4 we obtain for $I_{\text{shaft}} = 0$

$$\omega = \frac{\left(\frac{5 \text{ liters}}{s}\right) (\cos 30^\circ) (1000 \text{ mm}^2 / \text{m})^2}{3 (18 \text{ mm}^2) (1000 \text{ liters} / \text{m}^3) (0.5 \text{ m})} = 160 \frac{\text{rad}}{s}$$

(c) For $\theta = 60^\circ$

From Eq. 4 we obtain for $I_{\text{shaft}} = 0$

$$\omega = \frac{\left(\frac{5 \text{ liters}}{s}\right) (\cos 60^\circ) (1000 \text{ mm}^2 / \text{m})^2}{(3) (18 \text{ mm}^2) (1000 \text{ liters} / \text{m}^3) (0.5 \text{ m})} = 92.5 \frac{\text{rad}}{s}$$
5.70 Water flows through a valve (see Fig. P5.70) with a weight flowrate, \( \dot{m} \), of 1000 lb/s. The pressure just upstream of the valve is 90 psi, and the pressure drop across the valve is 5 psi. The inside diameters of the valve inlet and exit pipes are 12 and 24 in. If the flow through the valve occurs in a horizontal plane, determine the loss in available energy across the valve.

The control volume shown in the sketch above is used. We can use Eq. 5.54 to determine the loss in available energy associated with the incompressible, steady flow through this control volume. Thus

\[
\text{loss} = \frac{P_1 - P_2}{\rho} + \frac{V_2^2 - V_1^2}{2}
\]

From the conservation of mass principle

\[
V_1 = \frac{\dot{m}}{\rho A_1} = \frac{\dot{m}}{\rho \pi D_1^2/4}
\]

and

\[
V_2 = \frac{\dot{m}}{\rho \pi D_2^2/4}
\]

Thus

\[
\text{loss} = \frac{P_1 - P_2}{\rho} + \frac{\left(\frac{\dot{m}}{\rho \pi D_1^2/4}\right)^2}{2} \left(\frac{1}{D_1^4} - \frac{1}{D_2^4}\right)
\]

\[
\text{loss} = \frac{\left(5 \ \text{lb} \text{/in}^2 \right) (144 \text{ in}^3)}{1.94 \text{ slugs} \text{/ft}^2} \times \frac{1}{2} \left(\frac{1000 \ \text{lb} \text{/s}}{5 \ \text{ft}^2} \right)^2 \left(\frac{12 \text{ in}}{5 \text{ ft}}\right)^4 \left(\frac{12 \text{ in}}{24 \text{ in}}\right)^4 \left(\frac{16 \text{ lb}}{2 \text{ slugs}}\right)
\]

\[
\text{loss} = 560 \text{ ft} \cdot \text{lb} / \text{slug}
\]
Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.90. The loss in available energy associated with 2.5 ft/s being pumped from sections (1) to (2) is $61 \frac{V^2}{2}$ ft-lb/s, where $V$ is the average velocity of water in the 8-in. inside diameter piping involved. Determine the amount of shaft power required.

For the flow from section (1) to section (2) Eq. 5.56 leads to

$$W_{\text{shaft net in}} = \rho Q \left[ g (z_2 - z_1) + \text{loss} \right] = \rho Q \left[ g (z_2 - z_1) + 61 \frac{V^2}{2} \right] \quad (1)$$

From the volume flowrate we obtain

$$\bar{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{(2.5 \text{ ft}^3)}{s} = \frac{\pi / 8 \text{ in.}}{12 \text{ in.}} = 7.162 \frac{\text{ft}^3}{s}$$

Thus, from Eq. 1

$$W_{\text{shaft net in}} = (1.94 \text{ slugs} \frac{\text{ft}}{s}) (2.5 \frac{\text{ft}^3}{s}) \left[ \left(32.2 \frac{\text{ft}}{s^2} \right)(50 \text{ ft}) \right]$$

$$+ \left(61 \left(7.162 \frac{\text{ft}}{s} \right)^2 \right) \left(\frac{1 \text{ lb}}{\text{slug} \cdot \frac{\text{ft}^2}{s^2}} \right) \left(\frac{\text{ft-lb}}{550 \text{ ft-lb} \cdot \text{s/hp}} \right)$$

or

$$W_{\text{shaft net in}} = 28 \text{ hp}$$
Problem 5.92

(See Fluids in the News article titled “Curtain of air”)

Section 5.3.3.) The fan shown in Fig. P5.92 produces an air current to separate a loading dock from a cold storage room. The air curtain is 5 ft. high, 10 ft. wide, and 0.5 ft thick moving with speed \( V_0 = 72 \text{ ft/s}. \)

The air curtain serves to prevent a flow of air from entering the cold storage room. The cross section of the air curtain is shown in Fig. P5.92.

To produce this flow, \( h_s = \frac{1}{2} \rho V_0^2 = \frac{1}{2} (1.225 \text{slug/ft}^3)(72 \text{ ft/s})^2 = 288 \text{ ft-lbf/ft}^2. \)

Thus, \( h_s = h_c + \frac{V_0^2}{2g} = \frac{1}{2} \frac{V_0^2}{g} = \frac{1}{2} (9.8 \text{ ft/s}^2)(72 \text{ ft/s})^2 = 288 \text{ ft-lbf/ft}^2. \)

Hence,

\[
W_i = \frac{\rho A h_c}{h_s} = \frac{0.000338 \text{slug/ft}^2 \cdot 32.2 \text{ ft} \cdot 10 \text{ ft}}{288 \text{ ft-lbf/ft}^2} = 0.0105 \text{ hp}.
\]

\[
W_i = \frac{\rho A h_c}{h_s} = \frac{0.5 \text{ ft} \cdot 32.2 \text{ ft} \cdot 10 \text{ ft}}{288 \text{ ft-lbf/ft}^2} = 0.0105 \text{ hp}.
\]