Homework 6
Questions: 6.6, 6.22, 6.66, 6.76 and another question

0) Write a ½-1 page summary of the NSF video on fluid mechanics that you watched.

1) The three components of velocity in a flow field are given by

$$u=x^2+y^2+z^2$$
$$v=xy+yz+z^2$$
$$w=-3xz-z^2/2+4$$

Determine (a) the volumetric dilatation rate and interpret the results, (b) an expression for the rotation vector. Is this flow an irrotational flow field.

Comment: Make sure to look up the rotation vector in 3-d as it will have three components (ω_x, ω_y and ω_z), which are not all listed in the notes

2) It is proposed that a 2-d, incompressible flow field be described by the velocity components

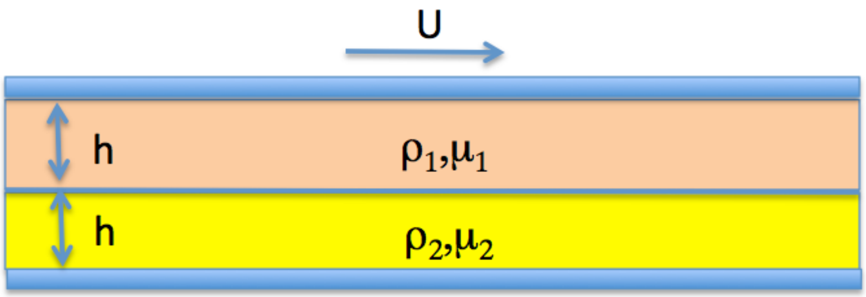
$$u=Ay \qquad \text{and} \qquad v=Bx$$

where A and B are positive constants. (a) Will the continuity equation be satisfied? (b) Is the flow irrotational?, (c) Determine the equation for the streamlines and show a sketch of the streamline that passes through the origin. Indicate the direction of flow along this streamline

3) Two fixed horizontal, parallel plates are spaced 0.2 inches apart. A viscous liquid ($\mu=8\text{x}10^{-3}$ lb s/ft2, SG=0.9) flows between the plates with a mean velocity of 0.9 ft/s. Determine the pressure-drop per unit length in the direction of flow. What is the maximum velocity in the channel?

4) Two immiscible, incompressible, viscous fluids having the same densities but different viscosities are contained between two infinite, horizontal, parallel plates (see figure below). The bottom plate is fixed and the upper plate moves with a constant velocity U. Determine the velocity at the interface. Express your answer in terms of U, μ_1 and μ_2. The motion of the fluid is caused entirely by the movement of the upper plat, i.e. there is no pressure gradient in the x-direction. The fluid velocity and shearing stress is continuous across the interface between the two fluids. Assume laminar flow.

Comment: Depart from the Navier-Stokes equations in each fluid (i.e. one set of N-S equations for fluid 1 and another for fluid 2) to solve this problem, neglecting terms and justifying why they can be neglected



Fixed plate

5) Flow Between Concentric Cylinders - In the figure below we depict two concentric cylinders. The inside one at radius R_{in} rotates at angular velocity ω_{in} and the outside one at radius R_{out} at angular velocity ω_{out}. There is a fluid of viscosity μ between R_{in} and R_{out}.

Departing from the two dimensional Navier-Stokes equations in cylindrical coordinates (in the r-theta plane only) derive the velocity profile between the two rotating cylinders (i.e. $R_{in} < r < R_{out}$.). State your assumptions and justify them clearly. Define your boundary conditions clearly too.

Hint*: The solution to the ODE* $x^2 \frac{d^2 f(x)}{dx^2} + x \frac{df(x)}{dx} - f(x) = 0$ *is* $f(x) = \frac{C_1}{x} + C_2 x$ *where* C_1 *and* C_2 *are constants. This is known as an Euler equation.*