7.12 At a sudden contraction in a pipe the diameter changes from $D_1$ to $D_2$. The pressure drop, $\Delta p$, which develops across the contraction is a function of $D_1$ and $D_2$, as well as the velocity, $V$, in the larger pipe, and the fluid density, $\rho$, and viscosity, $\mu$. Use $D_1$, $V$, and $\mu$ as repeating variables to determine a suitable set of dimensionless parameters. Why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

\[ \Delta p = f(D_1, D_2, V, \rho, \mu) \]

\[ \Delta p = \frac{FL^2}{D_1} \quad D_1 \equiv L \quad D_2 \equiv L \quad V \equiv LT^{-1} \quad \rho \equiv FL^{-4}T^2 \quad \mu \equiv FL^{-2}T \]

From the pi theorem, $6 - 3 = 3$ dimensionless parameters required. Use $D_1, V, \mu$ as repeating variables. Thus,

\[ \Pi_1 = \frac{\Delta p}{D_1} \frac{D_1^a}{V^b} \frac{\mu^c}{c = 1, b = -1, c = -1}, \text{and therefore} \]

\[ \Pi_1 = \frac{\Delta p D_1}{V \mu} \]

Check dimensions using MLT system:

\[ \frac{\Delta p}{V \mu} \frac{D_1^a}{(LT^{-1})(ML^{-1}T^{-1})} = M^0L^0T^0 \implies OK \]

For $\Pi_2$:

\[ \Pi_2 = \frac{D_2}{D_1} \frac{D_1^a}{V^b} \frac{\mu^c}{c = 0} \]

\[ 1 + a + b - 2c = 0 \quad (\text{for } F) \]

\[ -b + c = 0 \quad (\text{for } L) \]

It follows that $a = 1, b = 0, c = 0$, and therefore

\[ \Pi_2 = \frac{D_2}{D_1} \quad (\text{cont'ed}) \]
π₂ is obviously dimensionless.

For \( \Pi_3 \):

\[
\Pi_3 = \rho \frac{D_1^a V^b}{\mu^c} \quad (FL^{-4}T^{-2})(L)^a(LT^{-1})^b(FL^{-2}T)^c = FOL^0T^0
\]

\[
1 + c = 0 \quad \text{(for F)}
\]
\[
-4 + a + b - 2c = 0 \quad \text{(for L)}
\]
\[
2 - b + c = 0 \quad \text{(for T)}
\]

It follows that \( a = 1, b = 1, c = -1 \) and therefore

\[
\Pi_3 = \rho \frac{D_1 V}{\mu}
\]

Check dimensions using MLT system:

\[
\rho \frac{D_1 V}{\mu} \equiv \frac{(ML^{-3})(L)(LT^{-1})}{ML^{-1}T^{-1}} = M^0L^0T^0 \quad \therefore \text{OK}
\]

Thus,

\[
\frac{\Delta p}{p} \frac{D_1}{V} = \phi \left( \frac{D_2}{D_1} \right) \left( \frac{\rho D_1 V}{\mu} \right)
\]

From the continuity equation,

\[
V \frac{\Pi}{\Phi} D_1^2 = V_s \frac{\Pi}{\Phi} D_2^2
\]

Where \( V_s \) is the velocity in the smaller pipe. Since

\[
V_s = \left( \frac{D_1}{D_2} \right)^2 V
\]

\( V_s \) is not independent of \( D_1, D_2, \) and \( V \) and therefore should not be included as an independent variable.
7.14 Under certain conditions, wind blowing past a rectangular speed limit sign can cause the sign to oscillate with a frequency \( \omega \) (See Fig. P7.14 and Video V9.9.) Assume that \( \omega \) is a function of the sign width, \( b \), sign height, \( h \), wind velocity, \( V \), air density, \( \rho \), and an elastic constant, \( k \), for the supporting pole. The constant, \( k \), has dimensions of \( FL \). Develop a suitable set of pi terms for this problem.

\[
\omega = f(b, h, V, \rho, k)
\]

\[
\omega = T^{-1} b = L h = L V = LT^{-1} \rho = FL^{-1} T^{-2} k = FL
\]

From the \( \pi \) theorem \( 6-3 = 3 \) \( \pi \) terms required. Use \( b, V, \) and \( \rho \) as repeating variables. Thus,

\[
\pi_1 = \omega b^a V^b \rho^c
\]

and

\[
(T^{-1}) (L)^c (LT^{-1})^b (FL^{-1} T^{-2})^c = F^0 L^0 T^0
\]

so that

\[
c = 0 \quad (\text{for } F)
\]

\[
a + b - 4c = 0 \quad (\text{for } L)
\]

\[
-1 - b + 2c = 0 \quad (\text{for } T)
\]

It follows that \( a = 1, b = -1, c = 0 \), and therefore

\[
\pi_1 = \frac{\omega b}{V}
\]

Check dimensions:

\[
\frac{\omega b}{V} = \frac{(T^{-1})(L)}{(LT^{-1})} = L^0 T^0 \quad ; \quad \text{OK}
\]

For \( \pi_2 \):

\[
\pi_2 = h b^a V^b \rho^c
\]

\[
(L)(L)^a (LT^{-1})^b (FL^{-1} T^{-2})^c = F^0 L^0 T^0
\]

\[
c = 0 \quad (\text{for } F)
\]

\[
a + b - 4c = 0 \quad (\text{for } L)
\]

\[
-1 - b + 2c = 0 \quad (\text{for } T)
\]

It follows that \( a = -1, b = 0, c = 0 \), and therefore

\[
\pi_2 = \frac{h}{b}
\]

which is obviously dimensionless. (cont.)
For $\Pi_3$:

$$\Pi_3 = k b^a \sqrt{L^b \rho^c}$$

$$(FL)(L)^a(LT^{-1})^b(FL^{-1}T^2)^c = \rho^0 L^0 T^0$$

1 + c = 0 \quad \text{(for F)}

1 + a + b - 4c = 0 \quad \text{(for L)}

-b + 2c = 0 \quad \text{(for T)}

It follows that $a = -3$, $b = -2$, $c = -1$, and therefore

$$\Pi_3 = \frac{k}{b^3 \sqrt{L^2 \rho}}$$

Check dimensions using MLT system:

$$\frac{k}{b^3 \sqrt{L^2 \rho}} \equiv \frac{ML^2 T^{-2}}{(L^2)(LT^{-1})^2(ML^{-3})} = M^0 L^0 T^0 \quad \text{OK}$$

Thus,

$$\frac{\omega b}{V} = \phi \left( \frac{b}{b}, \frac{k}{b^3 \sqrt{L^2 \rho}} \right)$$

7-18
7.10 The buoyant force, $F_B$, acting on a body submerged in a fluid is a function of the specific weight, $\gamma$, of the fluid and the volume, $V$, of the body. Show, by dimensional analysis, that the buoyant force must be directly proportional to the specific weight.

\[ F_B = f(\gamma, V) \]

\[ F_B = F \quad \gamma = FL^{-3} \quad V = L^3 \]

From the pi theorem, $3-2 = 1$ pi term required.

By inspection:

\[ \Pi_1 = \frac{F_B}{\gamma V} \Rightarrow \frac{F}{(FL^{-3})(L^3)} = F^0L^0 \]

Check using MLT:

\[ \frac{F_B}{\gamma V} = \frac{MLT^{-2}}{(ML^{-2}T^{-2})(L^3)} = M^0L^0T^0 : \text{OK} \]

Since there is only 1 pi term, it follows that

\[ \frac{F_B}{\gamma V} = C \]

where $C$ is a constant. Thus,

\[ F_B = C \gamma V \]

and

\[ F_B \propto \gamma \]
A liquid flows with a velocity $V$ through a hole in the side of a large tank. Assume that

$$V = f(h, g, \rho, \sigma)$$

where $h$ is the depth of fluid above the hole, $g$ is the acceleration of gravity, $\rho$ the fluid density, and $\sigma$ the surface tension. The following data were obtained by changing $h$ and measuring $V$, with a fluid having a density $\rho = 10^3$ kg/m$^3$ and surface tension $\sigma = 0.074$ N/m.

$$V \equiv LT^{-1}, \quad A \equiv L, \quad g \equiv LT^{-2}, \quad \rho \equiv ML^{-3}T^{-2}, \quad \sigma \equiv FL^{-1}$$

From the pi theorem, $3 - 2 = 1$ pi terms required.

By inspection for $\Pi_1$ (containing $V$):

$$\Pi_1 = \frac{V}{\sqrt{g \cdot h}} = \frac{LT^{-1}}{(LT^{-2})^{1/2} \cdot (L)^{1/2}} = L^0 T^0$$

For $\Pi_2$ (Containing $\rho$ and $\sigma$):

$$\Pi_2 = \frac{\rho \cdot g \cdot h^2}{\sigma} = \frac{(ML^{-3})(LT^{-2})(L^2)}{FL^{-1}} = M^0 L^0 T^0$$

Check using $MLT$:

$$\frac{\rho \cdot g \cdot h^2}{\sigma} = \frac{(ML^{-3})(LT^{-2})(L^2)}{M^{-1} L^{-1} T^{-2}} = M^0 L^0 T^0 : \text{OK}$$

Thus,

$$\frac{V}{\sqrt{g \cdot h}} = f\left(\frac{\rho \cdot g \cdot h^2}{\sigma}\right)$$

For the data given:

<table>
<thead>
<tr>
<th>$\rho g h^2 / \sigma$</th>
<th>$3.3 \times 10^4$</th>
<th>$13.3 \times 10^4$</th>
<th>$29.8 \times 10^4$</th>
<th>$53.0 \times 10^4$</th>
<th>$82.9 \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V/\sqrt{gh}$</td>
<td>1.41</td>
<td>1.41</td>
<td>1.41</td>
<td>1.41</td>
<td>1.41</td>
</tr>
</tbody>
</table>

The graph and table show that $V/\sqrt{gh}$ is independent of $\rho g h^2 / \sigma$. Thus, the variables $\rho$ and $\sigma$ could have been omitted.
7.38 The drag, \( D \), on a sphere located in a pipe through which a fluid is flowing is to be determined experimentally (see Fig. P7.38). Assume that the drag is a function of the sphere diameter, \( d \), the pipe diameter, \( D \), the fluid velocity, \( V \), and the fluid density, \( \rho \). (a) What dimensionless parameters would you use for this problem? (b) Some experiments using water indicate that for \( d = 0.2 \) in., \( D = 0.5 \) in., and \( V = 2 \) ft/s, the drag is \( 1.5 \times 10^{-3} \) lb. If possible, estimate the drag on a sphere located in a 2-ft-diameter pipe through which water is flowing with a velocity of 6 ft/s. The sphere diameter is such that geometric similarity is maintained. If it is not possible, explain why not.

(a) \[ D = f(d, D, V, \rho) \]
\[ D = f \quad d = L \quad D = L \quad V = LT^{-1} \quad \rho = FL^{-3}T^{2} \]

From the pi theorem, \( 5-3 = 2 \) pi terms required, and a dimensional analysis yields
\[ \frac{D}{\rho V^{2}D^{2}} = \phi \left( \frac{d}{D} \right) \]

(b) The similarity requirement is
\[ \frac{d_m}{D_m} = \frac{d}{D} \]
so that
\[ 0.2 \text{ in.} = \frac{d}{D} \text{ ft} \]

and
\[ d = 0.8 \text{ ft} \] (required diameter).

Thus, the prediction equation is
\[ \frac{D}{\rho V^{2}D^{2}} = \frac{D_m}{\rho_m V_m^{2}D_m^{2}} \]

So that
\[ D = \frac{D_m}{D_m} \left( \frac{V}{V_m} \right)^{2} \left( \frac{D}{D_m} \right)^{2} \]

and with \( \rho = \rho_m \)
\[ D = \left( \frac{6 - \frac{d}{1.5}}{2 \frac{d}{2 \frac{d}} \frac{1}{0.5/2 \text{ ft}}} \right)^{2} \left( \frac{2 \frac{d}{1/2 \text{ ft}}} {1.5 \times 10^{-3} \text{ lb}} \right) ^{2} = 31.1 \text{ lb} \]

7-45