1.11

1.11 A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09BH^{3/2}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in ft^3/s when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

$$Q = 3.09\ B\, H^{3/2}$$

$$[L^3 T^{-1}] \doteq [3.09][L][L]^{3/2}$$

$$[L^3 T^{-1}] \doteq [3.09][L]^{5/2}$$

Since each term in the equation must have the same dimensions the constant 3.09 must have dimensions of $L^{1/2}T^{-1}$ and is therefore not dimensionless. No.

Since the constant has dimensions its value will change with a change in units. No.

11 h;

1.31

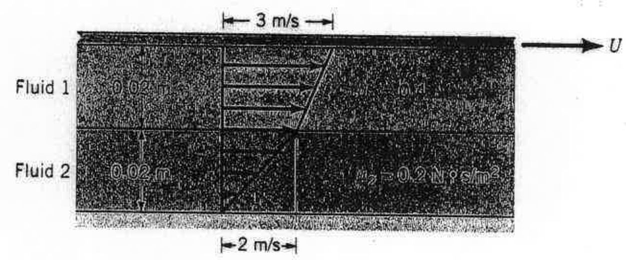
1.31 Nitrogen is compressed to a density of 4 kg/m³ under an absolute pressure of 400 kPa. Determine the temperature in degrees Celsius.

$$T = \frac{p}{\rho R} = \frac{400 \times 10^3 \ \frac{N}{m^2}}{\left(4 \ \frac{kg}{m^3}\right)\left(296.8 \ \frac{J}{kg \cdot K}\right)} = 337 \text{ K}$$

$$T_c = T_K - 273 = 337 \text{ K} - 273 = \underline{\underline{64 \ ^\circ C}}$$

1.45

1.45 As shown in **Video V1.4**, the "no-slip" condition means that a fluid "sticks" to a solid surface. This is true for both fixed and moving surfaces. Let two layers of fluid be dragged along by the motion of an upper plate as shown in Fig. P1.45. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the botton plate. Determine the ratio of these two shear stresses.



■ **FIGURE P1.45**

For fluid 1

$$\tau_1 = \mu_1 \left(\frac{du}{dy}\right)_{\text{top surface}} = \left(0.4 \frac{N \cdot s}{m^2}\right)\left(\frac{3\frac{m}{s} - 2\frac{m}{s}}{0.02\,m}\right) = 20 \frac{N}{m^2}$$

For fluid 2

$$\tau_2 = \mu_2 \left(\frac{du}{dy}\right)_{\text{bottom surface}} = \left(0.2 \frac{N \cdot s}{m^2}\right)\left(\frac{2\frac{m}{s}}{0.02\,m}\right) = 20 \frac{N}{m^2}$$

Thus,

$$\frac{\tau_{\text{top surface}}}{\tau_{\text{bottom surface}}} = \frac{20 \frac{N}{m^2}}{20 \frac{N}{m^2}} = \underline{\underline{1}}$$

45 br

1.61

1.61 Often the assumption is made that the flow of a certain fluid can be considered as incompressible flow if the density of the fluid changes by less than 2%. If air is flowing through a tube such that the air gage pressure at one section is 9.0 psi and at a downstream section it is 8.6 psi at the same temperature, do you think that this flow could be considered an incompressible flow? Support your answer with the necessary calculations. Assume standard atmospheric pressure.

For isothermal change in density

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}$$

so that

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}$$

The percent change in air densities between sections (1) & (2) is

$$\%\ change = \frac{\rho_1 - \rho_2}{\rho_1} \times 100$$

$$= \left(1 - \frac{\rho_2}{\rho_1}\right) \times 100 = \left(1 - \frac{p_2}{p_1}\right) \times 100$$

Thus,

$$\%\ Change = \left[1 - \frac{(8.6 + 14.7)\,psia}{(9.0 + 14.7)\,psia}\right] \times 100$$

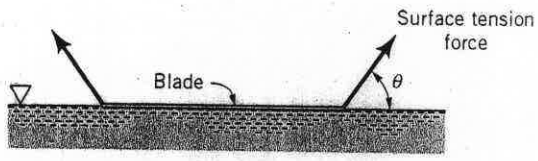
$$= 1.69\%$$

Since $1.69\% < 2\%$ the flow could be considered incompressible.
<u>Yes</u>.

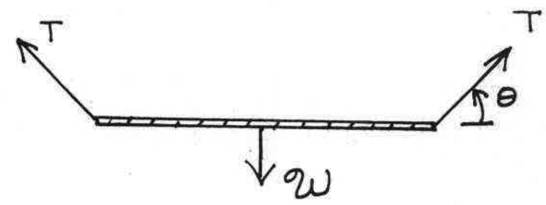
61 br

1.72

1.72 As shown in **Video V1.9**, surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water, but a single-edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in Fig. P1.72. **(a)** The mass of the double-edge blade is 0.64×10^{-3} kg, and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. **(b)** The mass of the single-edge blade is 2.61×10^{-3} kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.



■ **FIGURE P1.72**



(a) $\Sigma F_{vertical} = 0$

$$\mathcal{W} = T \sin\theta$$

where $\mathcal{W} = m_{blade} \times g$ and $T = \sigma \times$ length of sides.

$$\therefore \left(0.64 \times 10^{-3}\, kg\right)\left(9.81\, m/s^2\right) = \left(7.34 \times 10^{-2}\, \frac{N}{m}\right)\left(0.206\, m\right)\sin\theta$$

$$\sin\theta = 0.415$$

$$\underline{\underline{\theta = 24.5^\circ}}$$

(b) For single-edge blade

$$\mathcal{W} = m_{blade} \times g = \left(2.61 \times 10^{-3}\, kg\right)\left(9.81\, m/s^2\right)$$
$$= 0.0256\, N$$

and

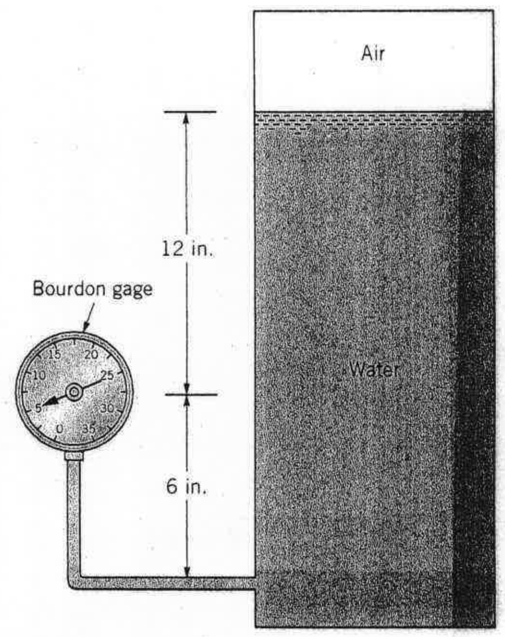
$$T \sin\theta = \left(\sigma \times \text{length of blade}\right)\sin\theta$$
$$= \left(7.34 \times 10^{-2}\, N/m\right)\left(0.154\, m\right)\sin\theta$$
$$= 0.0113 \sin\theta$$

In order for blade to "float" $\mathcal{W} < T\sin\theta$. Since maximum value for $\sin\theta$ is 1, it follows that $\mathcal{W} > T\sin\theta$ and single-edge blade will sink.

72 br

2.17

2.17 Bourdon gages (see **Video V2.3** and Fig. P2.17) are commonly used to measure pressure. When such a gage is attached to the closed water tank of Fig. P2.17 the gage reads 5 psi. What is the absolute air pressure in the tank? Assume standard atmospheric pressure of 14.7 psi.



■ FIGURE P2.17

$$p = \gamma h + p_0$$

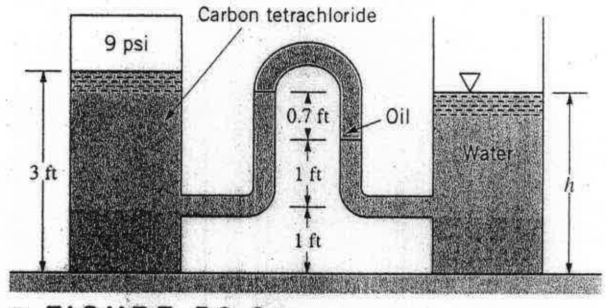
$$p_{gage} - \left(\frac{12}{12}\ ft\right)\gamma_{H_2O} = p_{air}$$

$$p_{air} = \left(5\ \frac{lb}{in.^2} + 14.7\ \frac{lb}{in.^2}\right) - \frac{(1\ ft)\left(62.4\ \frac{lb}{ft^3}\right)}{144\ \frac{in.^2}{ft^2}}$$

$$p_{air} = \underline{\underline{19.3\ psia}}$$

2.28

2.28 An inverted U-tube manometer containing oil ($SG = 0.8$) is located between two reservoirs as shown in Fig. P2.28. The reservoir on the left, which contains carbon tetrachloride, is closed and pressurized to 9 psi. the reservoir on the right contains water and is open to the atmosphere. With the given data, determine the depth of water, h, in the right reservoir.



■ **FIGURE P2.28**

Let p_A be the air pressure in left reservoir. Manometer equation can be written as

$$p_A + \gamma_{CCl_4}(3\,ft - 1\,ft - 1\,ft - 0.7\,ft) + \gamma_{oil}(0.7\,ft) - \gamma_{H_2O}(h - 1\,ft - 1\,ft) = 0$$

so that

$$h = \frac{p_A + \gamma_{CCl_4}(0.3\,ft) + \gamma_{oil}(0.7\,ft)}{\gamma_{H_2O}} + 2\,ft$$

$$= \frac{\left(9\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) + \left(99.5\,\frac{lb}{ft^3}\right)(0.3\,ft) + (0.8)\left(62.4\,\frac{lb}{ft^3}\right)(0.7\,ft)}{62.4\,\frac{lb}{ft^3}} + 2\,ft$$

$$= \underline{\underline{23.8\,ft}}$$

28 br

2.59

2.59 The concrete dam of Fig. P2.59 weighs 23.6 kN/m³ and rests on a solid foundation. Determine the minimum coefficient of friction between the dam and the foundation required to keep the dam from sliding at the water depth shown. Assume no fluid uplift pressure along the base. Base your analysis on a unit length of the dam.

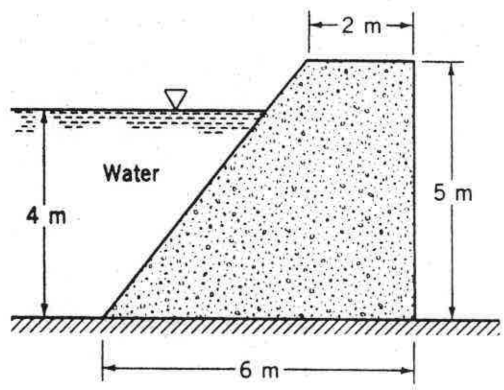
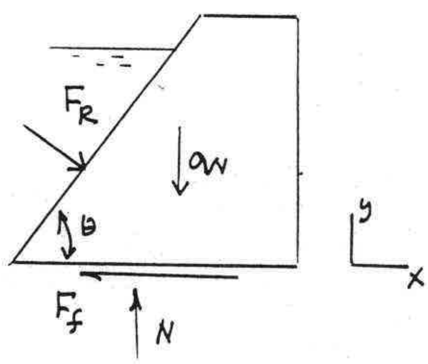


FIGURE P2.59



$$F_R = \gamma h_c A$$

where $A = \left(\frac{4\ m}{\sin 51.3°}\right)(1\ )$

$$\tan\theta = \frac{5\ m}{4\ m}$$

$$\theta = 51.3°$$

so that

$$F_R = \left(9.80\ \frac{kN}{m^3}\right)\left(\frac{4\ m}{2}\right)\left(\frac{4\ m}{\sin 51.3°}\right)(1\ m)$$

$$= 100\ kN$$

For equilibrium,

$$\Sigma F_x = 0$$

or

$$F_R \sin 51.3° = F_f = \eta N \quad \text{where } \eta \sim \text{coefficient of friction.}$$

Also, $\Sigma F_y = 0$

so that

$$N = \mathcal{W} + F_R \cos 51.3° \quad \text{where}$$

$$\mathcal{W} = (\gamma_{concrete})(\text{volume of concrete})$$

Thus,

$$N = \left(23.6\ \frac{kN}{m^3}\right)(20\ m^3) + (100\ kN)\cos 51.3° = 534\ kN$$

and

$$\eta = \frac{F_R \sin 51.3°}{N} = \frac{(100\ kN)\sin 51.3°}{534\ kN} = \underline{\underline{0.146}}$$

59 br

2.60*

2.60* Water backs up behind a concrete dam as shown in Fig. P2.60. Leakage under the foundation gives a pressure distribution under the dam as indicated. If the water depth, h, is too great, the dam will topple over about its toe (point A). For the dimensions given, determine the maximum water depth for the following widths of the dam: ℓ = 20, 30, 40, 50, and 60 ft. Base your analysis on a unit length of the dam. The specific weight of the concrete is 150 lb/ft^3.

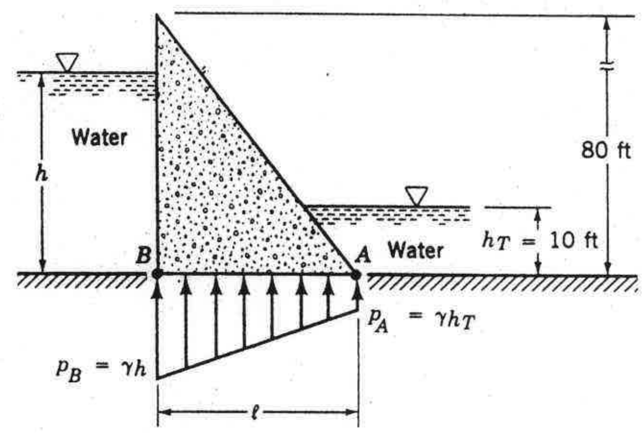
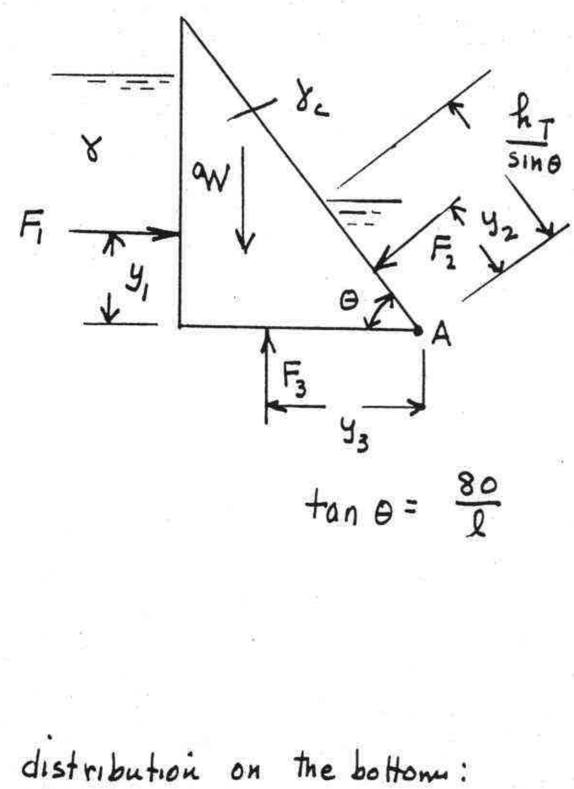


FIGURE P2.60

A free-body-diagram of the dam is shown in the figure at the right, where:



$F_1 = \dfrac{\gamma h^2}{2}$ (for unit length)

$\mathcal{W} = \gamma_c\left(\frac{1}{2}\right)(\ell)(80) = 40\gamma_c \ell$

$F_3 = \left(\dfrac{\gamma h + \gamma h_T}{2}\right)\ell$

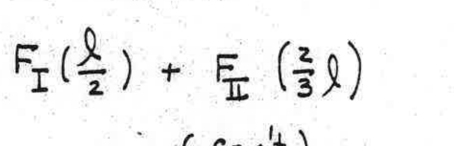
$F_2 = \gamma\left(\dfrac{h_T}{2}\right)\left(\dfrac{h_T}{\sin\theta}\right) = \dfrac{\gamma h_T^2}{2\sin\theta}$

$\tan\theta = \dfrac{80}{\ell}$

$y_1 = \dfrac{h}{3}$ $\qquad y_2 = \dfrac{1}{3}\left(\dfrac{h_T}{\sin\theta}\right)$

To determine y_3 consider the pressure distribution on the bottom:

$F_I = \gamma h_T \ell$



$F_{II} = \dfrac{\gamma}{2}(h - h_T)\ell$

Summing moments about A,

$$F_3 y_3 = F_I\left(\frac{\ell}{2}\right) + F_{II}\left(\frac{2}{3}\ell\right)$$

(con't)

60 br

2.60 * (Con't)

so that

$$y_3 = \frac{F_I\left(\frac{\ell}{2}\right) + F_{II}\left(\frac{2}{3}\ell\right)}{F_3}$$

where $F_3 = F_I + F_{II}$. Substitution of expressions for F_I and F_{II} yields,

$$y_3 = \frac{\ell\left(\frac{h_T}{3} + \frac{2}{3}h\right)}{h + h_T}$$

For equilibrium of the dam, $\Sigma M_A = 0$, so that

$$F_1 y_1 - \mathcal{W}\left(\tfrac{2}{3}\ell\right) - F_2 y_2 + F_3 y_3 = 0 \qquad (1)$$

and with $\gamma = 62.4\ lb/ft^3$, $\gamma_c = 150\ lb/ft^3$, and $h_T = 10\ ft$, then:

$F_1 = 31.2\, h^2 \qquad \mathcal{W} = 6000\,\ell \qquad F_2 = \dfrac{3120}{\sin\theta} \qquad y_2 = \dfrac{10/3}{\sin\theta}$

$F_3 = 31.2\,(h+10)\,\ell \qquad y_3 = \dfrac{\ell\left(\frac{10}{3} + \frac{2}{3}h\right)}{h + h_T} = \dfrac{(2h+10)\,\ell}{3(h+10)}$

Substitution of these expressions into Eq. (1) yields,

$$(31.2\,h^2)\left(\frac{h}{3}\right) - (6000\,\ell)\left(\frac{2}{3}\ell\right) - \left(\frac{3120}{\sin\theta}\right)\left(\frac{10/3}{\sin\theta}\right) + \left[31.2\,(h+10)\,\ell\right]\left[\frac{(2h+10)\,\ell}{3(h+10)}\right] = 0$$

which can be simplified to

$$\frac{31.2}{3}h^3 + 20.8\,\ell^2 h - 3896\,\ell^2 - \frac{10{,}400}{\sin^2\theta} = 0 \qquad (2)$$

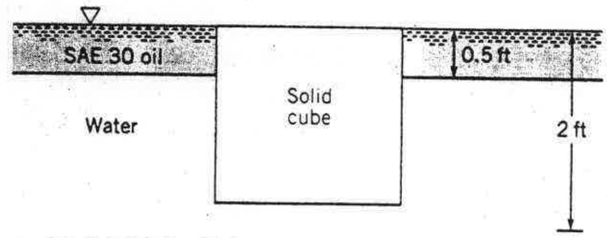
Thus, for a given ℓ, θ can be determined from the condition $\tan\theta = 80/\ell$, and Eq. (2) solved for h.

For the dam widths specified, the maximum water depths are given below. Note that for the two largest dam widths the water would overflow the dam before it would topple.

Dam width, ℓ, ft	Maximum depth, h, ft
20	48.2
30	61.1
40	71.8
50	81.1
60	89.1

2.73

2.73 A solid cube floats in water with a 0.5-ft thick oil layer on top as shown in Fig. P2.73. Determine the weight of the cube.



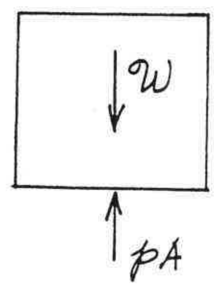
■ FIGURE P2.73

For equilibrium,

$$\sum F_{vertical} = 0$$

so that from free-body-diagram

$$\mathcal{W} = pA$$



where p is the pressure on bottom surface.

Thus,

$$p = \gamma_{oil}\,(0.5\,ft) + \gamma_{H_2O}\,(1.5\,ft)$$

$$= \left(57\,\frac{lb}{ft^3}\right)(0.5\,ft) + \left(62.4\,\frac{lb}{ft^3}\right)(1.5\,ft)$$

$$= 122\,\frac{lb}{ft^2}$$

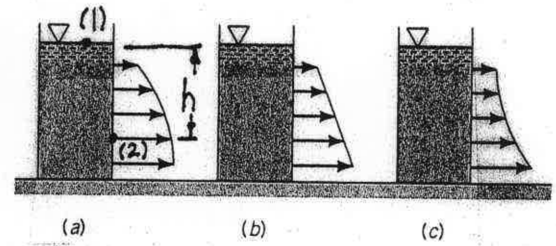
and

$$\mathcal{W} = \left(122\,\frac{lb}{ft^2}\right)(4\,ft^2) = \underline{\underline{488\,lb}}$$

73 br

3.27

3.27 Several holes are punched into a tin can as shown in Fig. P3.27 (See **Video V3.9**). Which of the figures represents the variation of the water velocity as it leaves the holes? Justify your choice.



■ FIGURE P3.27

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$ so that with $V_1 = 0$, $p_1 = 0$ and $z_1 = h_1$ at the free surface, then

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ or with $p_2 = 0$ (free jet) and $z_2 = h_2$

or

$$h_1 = \frac{V_2^2}{2g} + h_2$$ so that $V_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2gh}$

Thus,

or

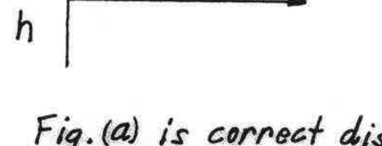
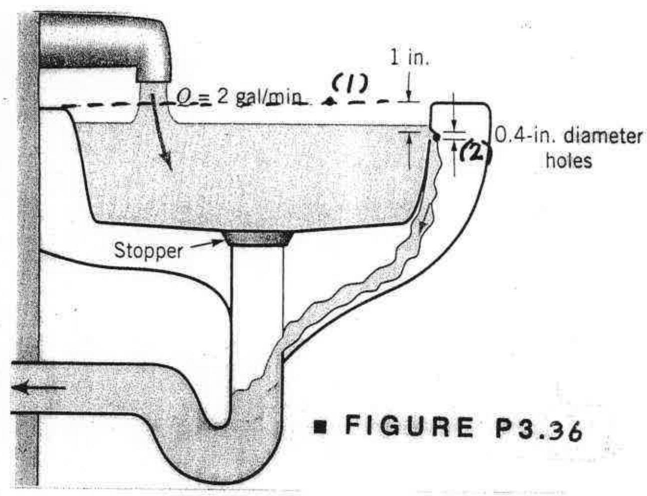


Fig. (a) is correct distribution

27 br

3.36

3.36 Water flows into the sink shown in Fig. P3.36 and **Video V5.1** at a rate of 2 gal/min. If the drain is closed, the water will eventually flow through the overflow drain holes rather than over the edge of the sink. How many 0.4-in.-diameter drain holes are needed to ensure that the water does not overflow the sink? Neglect viscous effects.



■ FIGURE P3.36

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{ , where } p_1 = 0,\ V_1 = 0, \text{ and } z_2 = 0,\ p_2 = 0$$

Thus,

$$z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad V_2 = \sqrt{2 g z_1} = \left[2\left(32.2\ \frac{ft}{s}\right)\left(\frac{1+0.2}{12}\ ft\right)\right]^{1/2} = 2.54\ \frac{ft}{s}$$

Also,

$$Q = n A_2 V_2 = n C_c \frac{\pi}{4} d_2^2 V_2 \text{ , where } n = \text{number of holes required,}$$

$d_2 = 0.4$ in., and C_c = contraction coef. $= 0.61$ (see Fig. 3.8)

Thus, with

$$Q = 2\ \frac{gal}{min}\left(\frac{1\ min}{60\ s}\right)\left(\frac{231\ in.^3}{1\ gal}\right)\left(\frac{1\ ft^3}{1728\ in.^3}\right) = 4.46 \times 10^{-3}\ \frac{ft^3}{s},$$

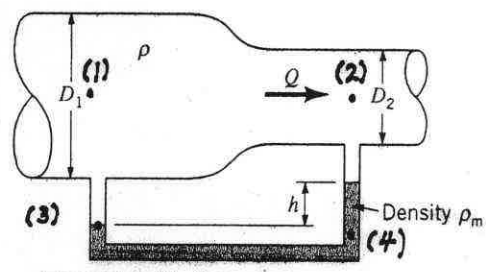
$$n = \frac{4Q}{\pi C_c d_2^2 V_2} = \frac{4\,(4.46 \times 10^{-3}\ ft^3/s)}{\pi (0.61)\left(\frac{0.4}{12}\right)^2 ft^2\,(2.54\ ft/s)} = 3.30$$

Thus, $\underline{\underline{4}}$ holes are needed.

36 br

3.47

3.47 An inviscid fluid flows steadily through the contraction shown in Fig. P3.47. Derive an expression for the fluid velocity at (2) in terms of D_1, D_2, ρ, ρ_m, and h if the flow is assumed incompressible.



■ FIGURE P3.47

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2$$

and $V_1 A_1 = V_2 A_2$ or $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$

Thus,

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2}{2g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \qquad (1)$$

but

$$p_3 = p_1 + \gamma \ell = p_4 = p_2 + \gamma(\ell - h) + \gamma_m h$$

or

$$p_1 - p_2 = \gamma \ell - \gamma h + \gamma_m h - \gamma \ell = (\gamma_m - \gamma)h = g(\rho_m - \rho)h$$

or

$$\frac{p_1 - p_2}{\gamma} = \left(\frac{\rho_m}{\rho} - 1\right)h \qquad (2)$$

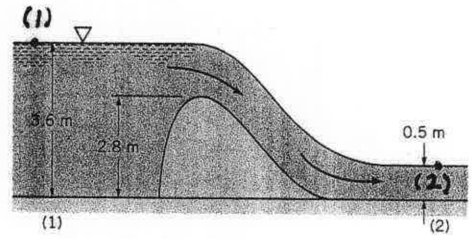
Combine Eqs. (1) and (2) to obtain

$$V_2 = \sqrt{\frac{2g(p_1 - p_2)/\gamma}{1 - \left(\frac{D_2}{D_1}\right)^4}} = \sqrt{\frac{2g\left(\frac{\rho_m}{\rho} - 1\right)h}{1 - \left(\frac{D_2}{D_1}\right)^4}}$$

47 br

3.71

3.71 Water flows over the spillway shown in Fig. P3.71. If the velocity is uniform at sections (1) and (2) and viscous effects are negligible, determine the flowrate per unit width of the spillway.



■ FIGURE P3.71

For points (1) and (2) on the free surface

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\,,\quad \text{where } p_1=0,\ p_2=0,\ z_1=3.6\,m$$
and $z_2=0.5\,m$

Also,

$$A_1V_1=A_2V_2\,,\quad \text{or}\quad V_2=\frac{A_1}{A_2}V_1=\frac{3.6\,m}{0.5\,m}V\quad \text{or}\quad V_2=7.2\,V_1$$

Hence,

$$\frac{V_1^2}{2g}+z_1=\frac{V_2^2}{2g}+z_2\quad \text{or}$$

$$\frac{V_1^2}{2g}+z_1=\frac{(7.2)^2V_1^2}{2g}+z_2$$

so that

$$V_1^2=2g(z_1-z_2)/[(7.2)^2-1]=2(9.81\tfrac{m}{s^2})(3.6m-0.5m)/[(7.2)^2-1]$$
$$=1.196\,\frac{m^2}{s^2}$$

Hence,

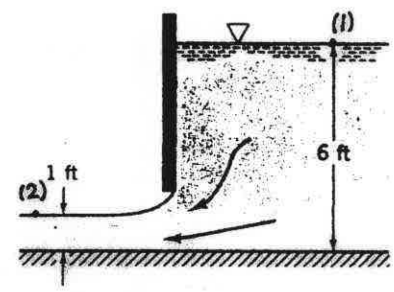
$V_1=1.09\,\frac{m}{s}$ so that for a $b=1\,m$ wide spillway,

$$Q=A_1V_1=(3.6m)(1m)(1.09\,\tfrac{m}{s})=\underline{\underline{3.92\,\frac{m^3}{s}}}$$

71 br

3.79

3.79 Water flows under the sluice gate shown in Fig. P3.79. Determine the flowrate if the gate is 8 ft wide.



■ FIGURE P3.79

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ p_2 = 0,\ z_1 = 6\text{ ft} \quad \text{and } z_2 = 1\text{ ft} \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{6\text{ ft}}{1\text{ ft}} V_1 = 6 V_1$$

Thus, Eq. (1) becomes

$$\left[6^2 - 1\right] V_1^2 = 2\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)(6-1)\text{ft} \quad \text{or} \quad V_1 = 3.03\ \frac{\text{ft}}{\text{s}}$$

Hence,

$$Q = A_1 V_1 = (6\text{ft})(8.0\text{ ft})\left(3.03\ \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{145\ \frac{\text{ft}^3}{\text{s}}}}$$

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