Question 1

(i) $F \sim v \rho A v \sim \rho A v^2$

$\therefore$ if we double velocity $\Rightarrow \rho A (2v)^2 \sim 4 \rho A v^2$

$\therefore \boxed{4F}$

(ii) $Q = vA$

A does not change $\Rightarrow 2Q = 2vA$

$\boxed{2v}$

(iii) $\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0$

$0 + (-\sin(y)) + \frac{\delta w}{d z} = 0$

$\frac{\delta w}{\delta z} = \sin(y) \Rightarrow w = z \sin(y)$

$\boxed{z \sin(y)}$

(iv) $\underset{\sim}{V} = \underset{\sim}{W} + \underset{\sim}{U}$

If V and U double, W doubles

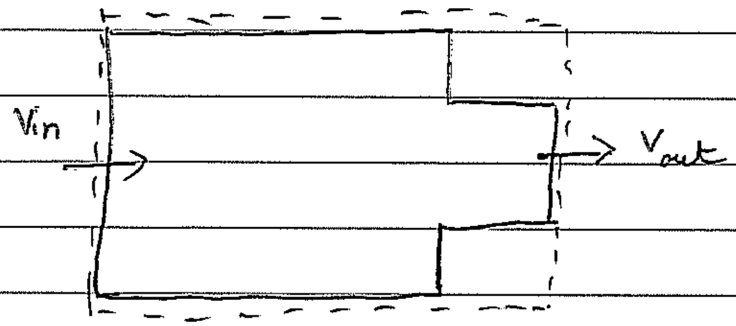
$\boxed{2W}$

(v) $u = \frac{\delta \psi}{\delta y}$ $\quad v = -\frac{\delta \psi}{\delta x}$

Incompressible conservation of volume $\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$

$$\underbrace{\frac{\delta^2 \psi}{\delta x \delta y} - \frac{\delta^2 \psi}{\delta x \delta y}}_{\boxed{\text{ALWAYS}}} = 0$$

Question 2



(i) $V_{in} A_{in} = V_{out} A_{out}$ Conservation of mass

Incompressible

$$\therefore V_{in} \frac{\pi D_{in}^2}{4} = V_{out} \frac{\pi D_{out}^2}{4}$$

$$D_{out} = \left(\frac{V_{in}}{V_{out}}\right)^{1/2} D_{in} = \left(\frac{8}{32}\right)^{1/2} 10 \text{ cm}$$

$$= \left(\frac{1}{4}\right)^{1/2} 10 \text{ cm}$$

$$= 5 \text{ cm}$$

(ii) Anchoring Force

Momentum Equation $\Rightarrow$ $\sum \underset{\sim}{V}_{out}\, \rho_{out} A_{out} V_{out} - \sum \underset{\sim}{V}_{in}\, \rho_{in} A_{in} V_{in} = \sum \underset{\sim}{F}$

X-direction only

Forces $\longrightarrow p_{in} A_{in}$

$\longleftarrow p_{out} A_{out} = 0$ (free jet)

$\longleftarrow F_x$ anchoring force

$$\therefore \quad V_{out}\, \rho_{out} A_{out} V_{out} - V_{in}\, \rho_{in} A_{in} V_{in} = p_{in} A_{in} - F_x$$

$$\therefore F_x = p_{in} A_{in} + \rho_{in} A_{in} V_{in}^2 - \rho_{out} A_{out} V_{out}^2$$

$$= (520 \times 10^3)\left(\frac{\pi (0.1)^2}{4}\right) + (1000)\left(\frac{\pi (0.1)^2}{4}\right) 8^2$$

$$- (1000)\left(\frac{\pi (0.05)^2}{4}\right) 32^2$$

$$= 2576.1 \text{ N}$$

Question 3

Energy Equation

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$P_1 = P_2 = 0$ (free surface and free jet)

$z_2 = 0$

$z_1 = 4m$

$V_1 = 0$ (free surface)

$V_2 = Q/A$

$h_s = \frac{20}{3} - \frac{4}{27} Q^2$

$h_L = \frac{4V^2}{2g} = \frac{4}{2gA^2} Q^2$

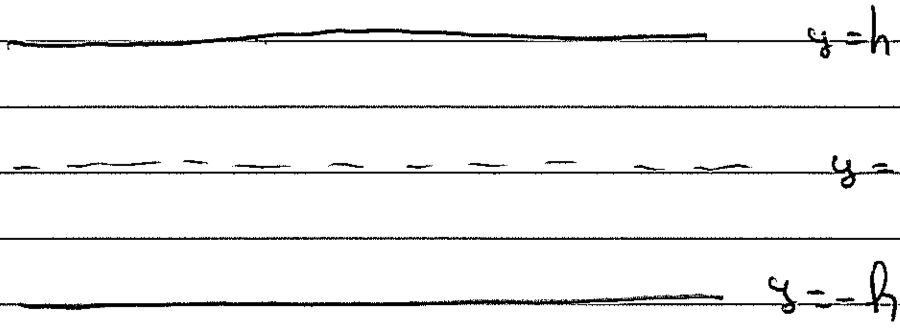
$$\therefore \quad 0 + 4 + 0 + \frac{20}{3} - \frac{4}{27} Q^2 - \frac{4}{2gA^2} Q^2 = 0 + 0 + \frac{1}{2gA^2} Q^2$$

$$\therefore \quad 4 + \frac{20}{3} = Q^2 \left(\frac{5}{2gA^2} + \frac{4}{27} \right)$$

$$Q = \left(\frac{4 + 20/3}{\frac{5}{2gA^2} + 4/27} \right)^{1/2} = 0.0647 \ m^3/s$$

Question 4

(1)



Assumptions : Incompressible
Steady
Fully Developed

Conservation of Mass : $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$

Using incompressible $\Rightarrow \underbrace{\cancel{\frac{\partial u}{\partial x}}}_{\text{fully developed}} + \frac{\partial v}{\partial y} = 0$

$\therefore \quad \frac{\partial v}{\partial y} = 0 \Rightarrow V = \text{const}$

$V = 0$ since $v = 0$ at boundary

$$X\text{-momentum} \Rightarrow \rho\left(\underbrace{\cancel{\frac{\delta u}{\delta t}}}_{\text{Steady}} + u\underbrace{\cancel{\frac{\delta u}{\delta x}}}_{\text{Fully Developed}} + \underbrace{\cancel{v\frac{\delta u}{\delta y}}}_{v=0}\right) = -\frac{\delta p}{\delta x} + \mu\underbrace{\cancel{\frac{\delta^2 u}{\delta x^2}}}_{\text{Fully Developed}} + \mu\frac{\delta^2 u}{\delta y^2}$$

$$\therefore \quad \mu\frac{\delta^2 u}{\delta y^2} = \frac{\delta p}{\delta x}$$

$$\Rightarrow \quad u = \frac{1}{\mu}\frac{\delta p}{\delta x}\frac{y^2}{2} + Ay + B$$

Boundary Conditions $\Rightarrow$ $u = 0$ @ $y = -h$

$u = 0$ @ $y = h$

$$\therefore \quad A = 0$$

and

$$B = \frac{-1}{\mu}\frac{\delta p}{\delta x}\,h^2/2$$

$$\Rightarrow \boxed{u(y) = \frac{1}{2\mu}\frac{\delta p}{\delta x}\left(y^2 - h^2\right)} \qquad \text{Q.E.D.}$$

(iii) $Q = \int v \, dA$

$$= \int_{-h}^{h} \frac{1}{2\mu} \frac{\delta p}{\delta x} (y^2 - h^2) \, dy \qquad (\text{unit width} \Rightarrow dA = 1 \times dy)$$

$$= \frac{1}{2\mu} \frac{\delta p}{\delta x} \Big[y^3/3 - y h^2 \Big]_{y=-h}^{y=h}$$

$$= \frac{1}{2\mu} \frac{\delta p}{\delta x} \Big[h^3/3 - h^3 + h^3/3 - h^3 \Big]$$

$$= \frac{1}{2\mu} \frac{\delta p}{\delta x} \Big[-\frac{4}{3} h^3 \Big]$$

$$= -\frac{2}{3\mu} \frac{\delta p}{\delta x} h^3$$

$$= \frac{-2}{3(1.12 \times 10^{-3})} (-1) (5 \times 10^{-3})^3$$

$$= 7.44 \times 10^{-5} \ m^3/s$$