Sample Final

Question 1 (50 points)

Multiple Choice – 10 questions, 5 points each

1. When fluid flow is characterized as fully turbulent, which of the following is a true statement?

✓Friction factor decreases with increasing Reynolds Number
Friction factor increases with increasing Reynolds Number
✓Friction factor is independent of Reynolds Number
Friction factor is independent of relative roughness

This was ambiguous and so both count

2. A thin rectangular plate has width w and height h. We want to calculate the drag D, which we assume is a function of width (w), height (h), kinematic viscosity (m), fluid density (r) and flow velocity (V). How many independent dimensionless Pi groups can we form?

1 2 (3) 4 5 6

3. The flow rate of water at room temperature in a pipe of 1 cm diameter is 0.1 m^3/s. The flow is

Laminar Transitional (Turbulent)

$Re = \frac{\rho U D}{\mu} \approx \frac{(1000)\frac{(0.1)}{(\pi/4)(10^{-2})^2}10^{-2}}{10^{-3}} \approx 10^5 >> 4000$

4. A model pipe is scaled down in diameter tenfold. Water is to be used in both. Dynamic similarity requires that the Reynolds number be the same in each. The velocity in the model should be

The same as the prototype
(10 times greater than prototype)
100 times greater than prototype
10 times smaller than prototype
100 times smaller than prototype

$\frac{U_m \rho_m L_m}{\mu_m} = \frac{U_p \rho_p L_p}{\mu_p}$

$U_m = U_p \frac{L_p}{L_m} = U_p\, 10$

5. When studying flow in a pipe

Major Losses are always greater than Minor Losses
Minor Losses depend very sensitively on the pipe roughness
✓Major Losses are reduced by decreasing pipe wall roughness
Bernoulli's equation is always valid

6. A jet with velocity V exiting from a pipe with diameter D causes a horizontal force, F, on a wall perpendicular to the jet. If the exiting velocity were to halve to 0.5V with the same pipe, the new force on the wall would be

$F \sim \rho A v^2$ (momentum eqn)

$(0.5)^2 \sim 1/4$

F/8 (F/4) $F/2^{1/2}$ F $2^{1/2}F$ 2F 4F 8F

7. A flow is incompressible and three-dimensional. The components in the x and y directions are u=3x, v=7y. What of the following is a correct value of w

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0 \Rightarrow \frac{\partial v}{\partial z} = -1$

(-10 z+3x^2) 10z 3z 7z 4z 21z

8. A dam has a water depth of h which produces a hydrostatic force F on the dam face. If the water depth doubles to 2h, the hydrostatic force on the dam face will then be

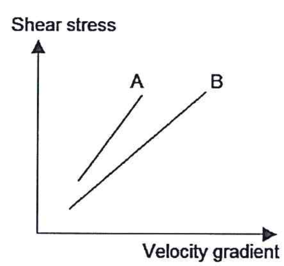
$F \sim \gamma h_c A$

$\sim \gamma \frac{h}{2}(wh) = \frac{\gamma w}{2} h^2$

[F/4] [F/2] [$F/2^{1/2}$]

[F] [$F*2^{1/2}$] [2F] ([4F])

9. The figure shows the relationship between shear stress and velocity gradient for two fluids, A and B. Based on this figure, which of the following is definitely true?



$\tau = \mu \frac{du}{dy}$

absolute viscosity

→ slope is bigger ⇒ μ bigger

✓Absolute viscosity of A is greater that that of B
Absolute viscosity of B is greater that that of A
Kinematic viscosity of A is greater that that of B
Kinematic viscosity of B is greater that that of A

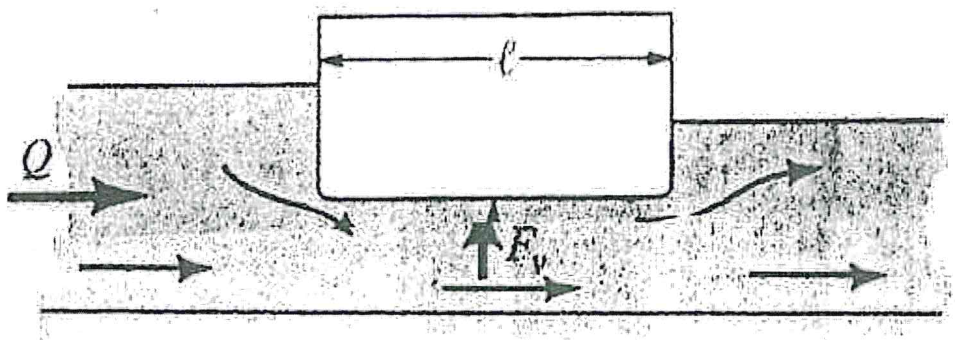
10. Which of these is true

Streamlines and streaklines are the same for any incompressible flow
Pathlines and streamlines are the same for any incompressible flow
✓Streamlines and streaklines are the same for any steady flow
Pathlines and streamlines are the same for any fully developed flow

Question 2 (25 points)

Water flowing under an obstacle is shown in the figure. The flow exerts a vertical force on the object, which is assumed to be a function of flow rate, density of water, acceleration of gravity, and the length of the object. A 1/20 scale model is to be used to predict the vertical force on the prototype

(a) Perform a dimensional analysis of the problem
(b) If the prototype flow rate is 100 m^3/s determine the flow rate for the model for the flows to be similar
(c) If the model force is F=100N, predict the corresponding force on the prototype



(a)
$[F] = MLT^{-2}$
$[Q] = L^3T^{-3}$
$[g] = LT^{-2}$
$[\rho] = ML^{-3}$
$[\ell] = L$

$\Pi_1 = \frac{F}{\rho \ell^3 g}$ $\quad \Pi_2 = \frac{Q}{\sqrt{g\ell^5}}$

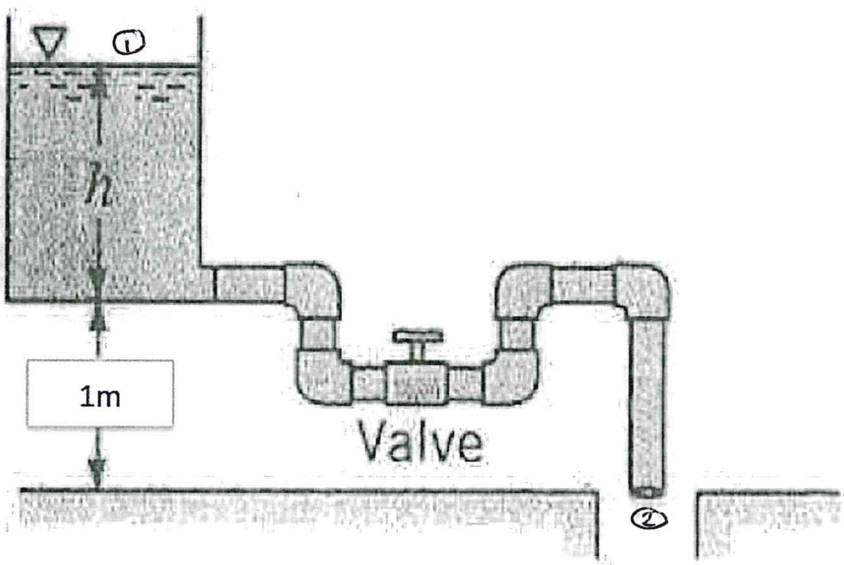
$\therefore \frac{F}{\rho g \ell^3} = \phi\left(\frac{Q}{\sqrt{g\ell^5}}\right)$

(b) $\frac{Q_m}{\sqrt{g_m \ell_m^5}} = \frac{Q_p}{\sqrt{g_p \ell_p^5}} \Rightarrow Q_m = Q_p\left(\frac{\ell_m}{\ell_p}\right)^{5/2} = 100\left(\frac{1}{20}\right)^{5/2}$
$= 0.056 \ m^3/s$

(c) $\frac{F_m}{\rho_m \ell_m^3 g_m} = \frac{F_p}{\rho_p \ell_p^3 g_p} \Rightarrow F_p = F_m\left(\frac{\ell_p}{\ell_m}\right)^3 = (100)(20)^3$
$= 800 \ kN$

Question 3 (25 points)

Water flows from the tank shown in the figure and the water depth of the tank in meters is given by h. The total length of the pipe is 7 meters and has diameter 2cm with a friction factor of 0.03. The loss coefficients for the entrance are 0.5, each elbow 1.5 and 10 for the valve. Calculate the flow rate when h=0.5m. If the tank has a cross section area of 1m^2, what is the rate at which the water depth decreases in the tank (i.e. the velocity of the interface – if you ignored it, is it reasonable to ignore)?



$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 - h_L = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$P_1 = P_2 = 0$

$v_1 = 0$

$z_2 = 0$

$z_1 = 1 + h$

$h_L = \left(f\frac{\ell}{D} + \sum K_i\right) v^2/2g$

$V_2 = V$

$$\Rightarrow z_1 = \left(f\frac{\ell}{D} + \sum K_i + 1\right)\frac{v^2}{2g}$$

$$1+h = \left(0.03\frac{7}{(2\times10^{-2})} + (0.5 + 5(1.5) + 10) + 1\right)\frac{v^2}{2g}$$

$$V = \left(\frac{(1+h)\,2g}{\left(f\frac{\ell}{D} + \sum K + 1\right)}\right)^{1/2} = 1 \text{ m/s}$$

$$v_1 A_1 = v_2 A_2 \Rightarrow v_1 = v_2\left(\frac{A_2}{A_1}\right) = 1\left(\frac{\pi/4\,(2\times10^{-2})^2}{1}\right) = 3.1415\times10^{-4} \text{ m/s}$$

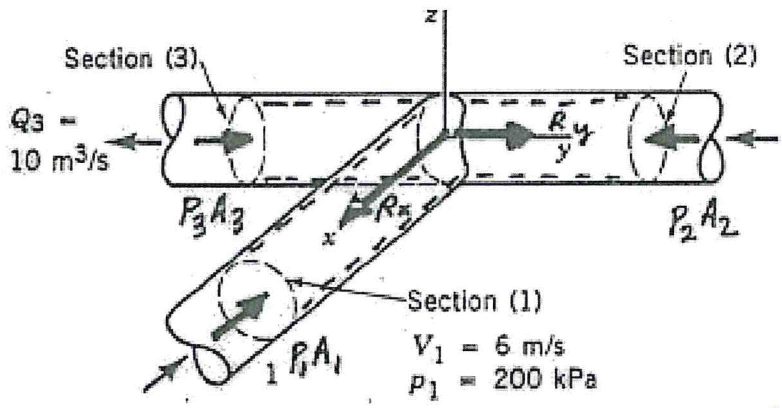
↓ tiny — reasonable to ignore

$$\left.\begin{aligned} v_1^2 &\approx 10^{-7} \\ v_2^2 &= 1 \end{aligned}\right\} \Rightarrow v_1^2 << v_2^2$$

Question 4 (25 points)

Assuming frictionless, incompressible 1-d flow of water through the horizontal tee as shown in the figure, estimate the x and y component of the force exerted by the tee on the water. Each pipe has a diameter of 1m.

As the flow is incompressible and frictionless you may use Bernoulli's equation to help you.



Momentum x $\quad R_x - p_1 A_1 = v_1 \rho A_1 v_1$ (note that this is $-(-v_1)\rho A_1 v_1$)

y $\quad R_y - p_2 A_2 + p_3 A_3 = v_2 \rho A_2 v_2 - v_3 \rho A_3 v_3$ (again $-(-v_2)\rho A_2 v_2$)

Conservation of Mass $\quad v_1 A_1 + v_2 A_2 = v_3 A_3 \Rightarrow v_2 = \frac{Q_3 - v_1 A_1}{A_2} = 6.7\ m/s$

$v_3 = \frac{Q_3}{A_3} = 12.7\ m/s$

Bernoulli $\Rightarrow \quad p_1 + \frac{1}{2}\rho v_1^2 = p_3 + \frac{1}{2}\rho v_3^2 = p_2 + \frac{1}{2}\rho v_2^2 \quad (z_1 = z_2 = z_3)$

$\therefore \quad p_3 = p_1 + \frac{1}{2}\rho(v_1^2 - v_3^2) = 137\ kPa$

$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 195.3\ kPa$

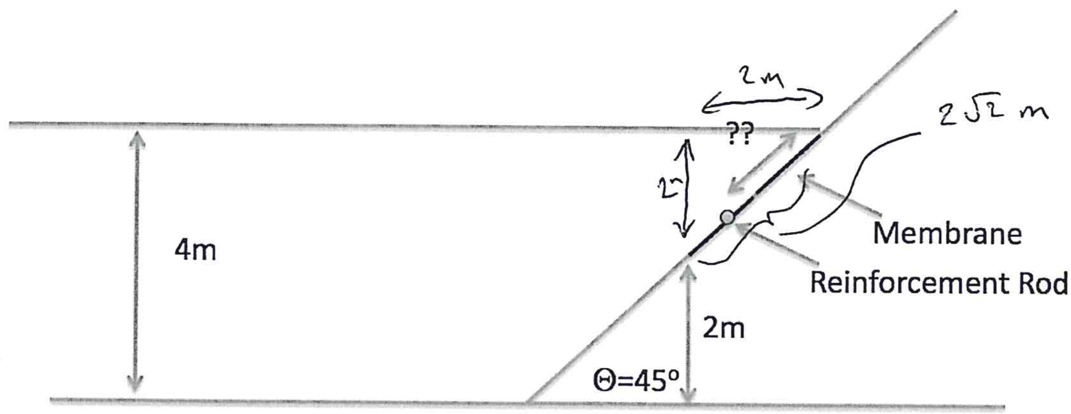
Now we know everything and can sub in above $\Rightarrow$

$R_x = p_1 A_1 + \rho A_1 v_1^2 = 185\ kN$

$R_y = p_2 A_2 - p_3 A_3 + v_2^2 \rho A_2 - v_3^2 \rho A_3 = -45.8\ kN$

Question 5 (25 points)

At a water treatment facility, a rectangular membrane of a certain width lies along the sloping boundary as depicted. The force on the membrane is not to exceed 100 kN. If the fluid in the reservoir is water, what is the maximum allowable width of the membrane? If the fluid were a very salty brine (SG=1.2) as may be used for desalination what would it be? A reinforced bar is to be placed across the membrane at the depth of the effective hydrostatic force. Where will this be? (i.e. what is the distance marked with ?? in the figure)



(i)

$$F_{eff} = \gamma h_c A$$
$$= (10^4)(1)(2\sqrt{2}\, w)$$

$$F_{eff\,max} = 100\text{ kN} \Rightarrow w = \frac{100000}{(10^4)(2\sqrt{2})} = \frac{5}{\sqrt{2}}\text{ m} = 3.5\text{ m}$$

(ii)

$$F_{eff} = \gamma h_c A \Rightarrow w = \frac{100000}{(1.2\times10^4)(2\sqrt{2})} = \frac{5}{1.2\sqrt{2}}\text{ m} = 2.94\text{ m}$$
$$= 1.2\, \gamma_{water} h_c A$$

(iii)

$$y_{eff} = \frac{\int_{y_e}^{y_u} y^2 dy}{\int_{y_e}^{y_u} y\, dy} = \frac{2}{3}\left(\frac{y_u^3 - y_e^3}{y_u^2 - y_e^2}\right) = \frac{2}{3}\left(\frac{(2\sqrt{2})^3 - 0^3}{(2\sqrt{2})^2 - 0^2}\right) = \frac{4\sqrt{2}}{3}$$

$$= 1.8856\text{ m}$$